ARE FRENCH METROPOLITAN AREAS

EVOLVING AS SELF-ORGANIZING SYSTEMS ?

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ABSTRACT

Peter Allen's Intra-urban model is a very appealing application of
bifurcation theory for simulating the evolution of an urban spatial struc-
ture. It is actually a spatial dynamic model, and it brings together many
well-known empirical regularities and well-established theoretic proposals,
as logistic growth, economic base theory, distance-decay functions, urban
ecology and actor's behaviour in an urban context. Until now, this model
has only been tested in fictitious urban situations and did prove its abi-
lity to simulate various urban evolutions, especially of the north-american
type. However, it needs to be tested in real-world situations. Main ques-
tions are : Until which extent the same set of equations is able to simula-
te various observed urban evolution ; and how many changes in parameters'
values are necessary to reproduce observed evolution in different towns. So
we tried to apply the model to a sample of french metropolitan areas.

The INTRA URBAN model was drawn up by researchers of the "Brussel's
school" in systems dynamic [1] [2]. It is one of the first models of bifur-
cation-type designed to describe the dynamics of spatial systems. That kind
of model use equations which were first established to simulate auto-orga-
nization phenomena in open physical systems situated far from equilibrium.
These models allow two kinds of changes in the system : progressive evolu-
tion along deterministic trajectories defined by non-linear equations ; and
sudden bifurcations, or change in trajectories, which may be produced by
random fluctuations. These bifurcations alter the structure of the system.
Such a conception of change is rather attractive for social sciences [3]
and seems to open new ways to the study of urban dynamics.

Inded, two series of models already designed and presented by P. Allen

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have proved their ability to reproduce theoretical and plausible long-range
or medium-range evolutions of fictitious spatial systems : on the one hand
by simulating the development of a central places network [4] ; and on the
other hand by generating various internal configurations of economic acti-
vities and residences in a large metropolitan area [2].

Before recording particular problems encountered and preliminary re-
sults in our tentative applications, we shall first recall the main specifi-
cities and mathematical structure of the INTRA URBAN model.

I - A DYNAMIC MODEL FOR INTRA-URBAN SPATIAL STRUCTURES

One characteristic of INTRA-URBAN model, of particular interest for
geographers, is that it is basically a dynamic and spatial model, which is a
great advantage comparing for example to FORRESTER's urban dynamics model.
In ALLEN's model, urban space is considered as an open system, made up of
about twenty parts (urban districts or communes). Each town-part is compe-
ting with each other for attracting jobs and residents. Each town-part is
characterized by :
- a number of jobs in different economic activities (industry, exporting
  tertiary, short-range and medium-range tertiary activities) ;
- a number of resident population having different occupational status
  (blue collars and white collars) ;
- a location respective to other town-parts and accessibility indices to
  transportation and information networks.

Starting from a given initial configuration, the model calculates
variations in the number of jobs and residents of each kind in each town-
part, according to spatial and non-spatial interactions between them.

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Contrary to its apparent over-simplification of urban structure, the
model recaptures the high complexity of urban dynamics by defining these
interactions. They are designed as mathematical relationships between va-
riables or parameters standing for empirical regularities, theoretical hy-
pothesis or conceptual entities often encountered in the litterature about
spatial dynamics, but not having yet been put all together in a single mo-
del. INTRA URBAN model thus incorporates :
- a logistic type model of growth for jobs and population of each kind in
  every town-part ;
- the principles of economic base theory, dividing jobs between exporting
  activities (the growth of which depends upon an external demand) and non-
  exporting activities (the growth of which depends upon local demand, by
  means of induction rates) ;
- a distance-decreasing-interaction model, of the exponential type with
  intervening opportunities, for people working in a town-part and choosing
  a residence location ;
- some principles of location theory for economic activities, including an
  "internal cooperativity" parameter standing for externalities and simila-
  rities in locational preferencies of each activity sector, and a "sensi-
  tivity to crowding" parameter figuring unequal needs in floorspace accor-
  ding to the nature of economic activity, competition for space, and satu-
  ration effects ;
- The model also incorporates hypothesis about actor behaviour in an urban
  context, for example grouping and segregative tendencies inside and bet-
  ween social groups ; information theory is requested to introduce some
  delay in reaction of entrepreneurs or individuals to a potential demand

and to define a parameter quoting for unequal information diffusion about quality of places among urban actors.

## II - DESCRIPTION OF THE EQUATIONS

Each equation describes the variation, per unit of time, of the number of jobs or residents in a town-part j. Each equation takes the following form :

$$\frac{dy_j}{dt} = a y_j (K - y_j)$$

where $y_j$ figures the number of jobs or residents already located in zone j, a (settled for $\epsilon$ or $\eta$) is a speed of reaction to offer or to demand, and K is a potential demand which determines the magnitude of the variation. (See tables 1 and 2 for the significance of parameters). This variation is then first a function of preexisting quantities of jobs or residents in considered town-part, which are growing according to a logistic curve.

1 - <u>For activities depending upon an external demand</u> (equation (1), industry and exporting tertiary), this variation is also a function :

- of external demand D ;

- of relative attractivity of each zone j for this kind of activity ; it is a ratio between intrinsec attractivity of zone j and the sum of all attractivities of zones j. The attractivity of a zone j is proportional to:

  - the volume of jobs of the same kind already present in that zone ;

  - the internal cooperativity of the activity being considered ( $\rho$ parameter) ;

  - the sensitivity to crowding ( $\tau$ ) ;

and inversely proportional to :

- production cost $\mu$

Table 1 : Equations of INTRA-URBAN models.

1 - Exporting activities

$$\frac{dS_j^E}{dt} = \epsilon^E S_j^E \left( D^E \frac{A_j}{\sum A_{j'}} - S_j^E \right)$$

with $A_j = \left[ \frac{(1 + e^E S_j^E)}{\mu^E + \alpha_j \phi^E} \cdot \frac{\tau^E}{\tau^E + \sum_{\ell'} x_j^{k'} + \sum_{\ell'} S_j^{\ell'}} \right]^{c_0^E}$

$S_j^E$ = number of employments of activity E in zone j

E = 1 = industry

E = 2 = exporting tertiary

2 - Short-range and medium-range tertiary functions

$$\frac{dS_j^u}{dt} = \epsilon^u S_j^u \left( \sum_{j'} \sum_{\ell'} \frac{\beta_\ell^u \cdot x_{j'}^{k'}}{(\mu^u + \phi^u \delta_{jj'})^{c_u}} \cdot \frac{A_{jj'}}{\sum_{j''} A_{j''j'}} - S_j^u \right)$$

with $A_{jj'} = \left[ \frac{1 + \sum_{\ell'} e^{\ell'} S_j^{\ell'}}{\mu^u + \phi^u \delta_{jj'}} \cdot \frac{\tau^u}{\tau^u + \sum_{k'} x_j^{k'} + \sum_{\ell'} S_j^{\ell'}} \right]^{c_0^u}$

$S_j^u$ = number of employments of function u in zone j

u = 3 = short-range function (local tertiary)

u = 4 = medium-range function (regional tertiary)

3 - Resident population

$$dx_j^k = \eta^k x_j^k \left[ \sum_{j'} \left( \sum_{\ell'} z^{k\ell'} S_j^{\ell'} \frac{R_{jj'}}{\sum_{j''} R_{j''j'}} \right) - x_j^k \right]$$

with $R_{jj'} = \left[ \frac{\nu^k (1 + \sigma^k x_j^k)}{\nu^k + \sum_{k'} x_j^{k'} + \sum_{\ell'} S_j^{\ell'}} e^{-\theta^k \delta_{jj'}} \right]$

$x_j^k$ = number of residents of category k in zone j

k = 1 = blue collars

k = 2 = white collars

j j' = distance between j and j'

for the significance of parameters, see table 2.

Table 2 : Significance of parameters

| Symbol | Significance | Values in first simulation |
|---|---|---|
| $\alpha$ (17,2) | For each zone, accessibility index to transportation and information network | .1 to 1 |
| b (2) | Distance-decay parameter for each resident population category | .5 .3 |
| $\beta$ (2,2) | Induction rate of short range and medium range tertiary function by residents of each category | .5 .3 .5 .3 |
| Ci (2) | Elasticity of demand for the cost of each service | 2. 2. |
| Co (4) | Information available to the actors | 10 – 10 – 10 – 10 |
| $\epsilon$ (4) | Speed of reaction of entrepreneurs to the demand | .001 .001 .01 .01 |
| $\eta$ (2) | Speed of adjustment of resident population to employment offer | .005 .005 |
| $\emptyset$ (4) | Transportation cost for each activity | .02 .0001 .025 .005 |
| $\mu$ (4) | Production cost for each activity | 1 1 1 1 |
| $\nu$ (2) | Sensitivity to crowding for residents | 1000 500 |
| $\rho$ (4) | Internal cooperativity for each activity sector | .01 .01 .0011 .0011 |
| $\sigma$ (2) | Grouping tendency for a category of residents | .2 .2 |
| $\tau$ (4) | Sensitivity to crowding for each activity | 1000 – 3000 – 1000 – 2000 |
| Z (2,4) | Share of each category of residents working in each activity sector | 1. .3 .7 .5 .5 .5 .5 |
| D (2) | External demand | 500 – 350 – |

- transportation cost $\emptyset$ weighted by accessibility index of zone j to transportation network $\alpha$j ;
- saturation of space in town-part being considered, as measured by total number of jobs and residents in that zone.

Each attractivity is exponentiated by the mean of a CO parameter which figures the level of unanimity in territorial preferences among urban actors.

2 – <u>For short-range and medium-range tertiary activities</u> (equation (2), local and regional services), employment variation is a function of internal demand emanating from each zone.

Internal demand for a type of services (function u ) in a zone j' is proportional to the total number of resident population, weighted by the mean level of demand per inhabitant for this kind of activity (induction rate $\beta$ ), and inversely proportional to the cost of this service. Cost is affected with an exponent Ci which figures degree of elasticity of services demand according to their cost. Cost incorporates production cost $\mu$ and transportation cost $\emptyset$, weighted by the distance $\delta$jj' between place of production j and place j' where consumers are living. This potential demand for services from zone j' to zone j is weighted by relative attractivity of zone j for zone j' (it is the ratio between attractivity of j for j' and sum of attractivities for j' from all town-parts).

Attractivity of a zone j for a zone j' is proportional to :
- the volume of preexisting jobs of type u in j ;
- the internal cooperativity of function u ($\rho$ ) ;
- the intensity of the crowding supported by the function u ;

and inversely proportional to :

- production cost $\mu$ ;

- transportation cost $\emptyset$ weighted by the distance $\delta jj'$ between zone j and
  zone j' ;

- crowding in zone j as measured by total number of employments and resi-
  dents already located at j.

Each attractivity is exponentiated by the mean of a Co parameter
standing for degree of uniformity in territorial preferences among the
actors.

3 - For the two categories of residents (equation (3), blue collars and
white collars), variation in a zone j is function of the number of resi-
dents of each category employed by the four economic activities in every
other town-part j', weighted by residential attractivity Rjj' of zone j
for people working in zones j'. Residential attractivity is a ratio between
intrinsic attractivity of zone j upon labor force working in j' and the
sum of residential attractivity of all town-parts.

Residential attractivity of a zone j for a category of people working
in a zone j' is proportional to :

- number of residents of this category already present in j ;

- affinity between members of a population of the same type $\sigma$) ;

- intensity of crowding supported by this category of residents $\nu$ .

Residential attractivity decreases :

- proportionally with spatial crowding as measured by total number of em-
  ployment and residents in j ;

- exponentially with distance between residential place j and working place
  j', according to a gradient b which measures sensitivity of each catego-
  ry to distant commuting.

The program realizes integration of each variable by an iterative
procedure. It is possible to choose the time-lag for calculation and dura-
tion of the simulation. Finally, the program allows for introduction, at
various stages of the simulation, of random fluctuations in the variable
values.

III - APPLICATION

For the first test of the model, we choosed the urban agglomeration
of Rouen. It offered sufficient similarities with the fictitious metropoli-
tan area used by P. Allen for theoretical simulations : a large urban area
(400.000 inhabitants), divided between 17 communes, the commune of Rouen
being in a central position and being more than ten times larger by resi-
dents and employments number than other communes of the agglomeration. Hea-
vy industry is mainly located, as in the theoretical case, along a river,
the Seine.

1 - The data

For each of the 17 communes, we collected comparable data for 1954,
1962, 1968 and 1975 (dates of censuses). Labor force is registred at work-
place and had to be divided between four types of economic activity. Exis-
ting nomenclature allows only direct calculation for number of people em-
ployed in industry. To distinguish three levels (local, regional and expor-
ting) among tertiary activities, we applied the minimum requirement tech-
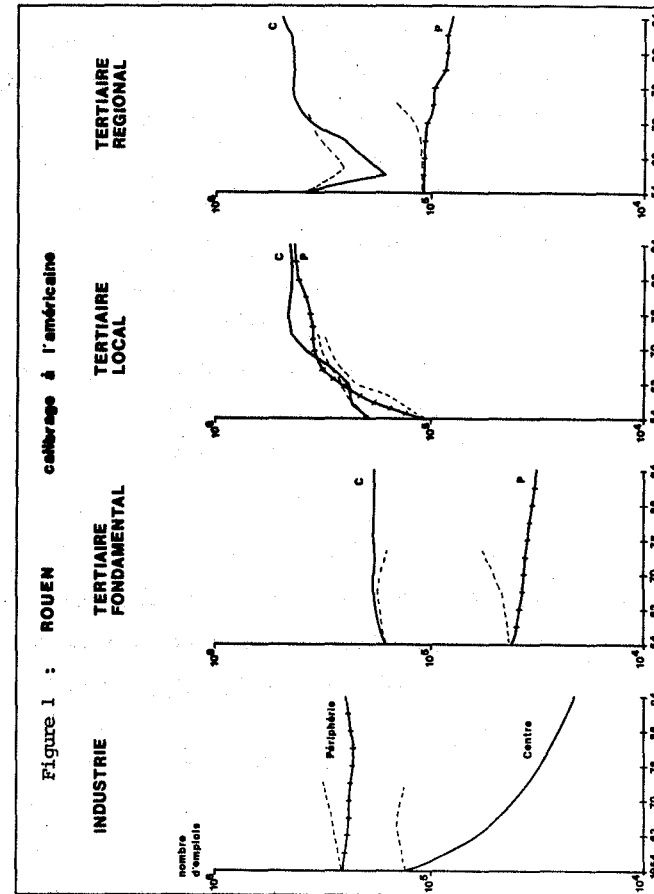nique. The two categories of resident population also stemmed from census

data (socio-professional nomenclature), by grouping all workers as "blue

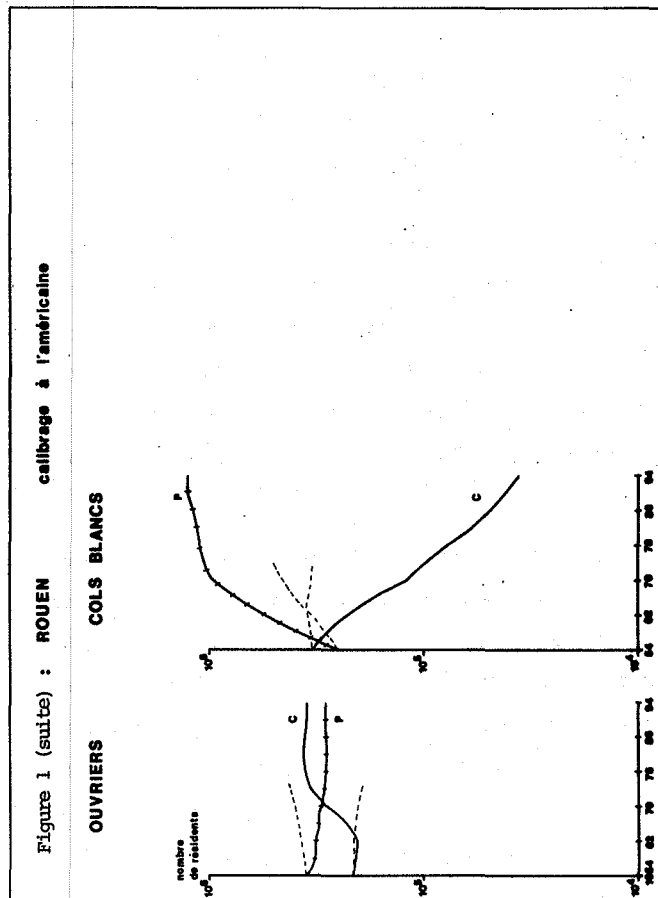collars" and all others as "white collars".

Relative location of the 17 communes are figured by geographic coor-

dinates of their centre of gravity. Their accessibility index for communi-

cation lines needed by industry was quantified by separating communes si-

tuated along the Seine river ( $\alpha$(j,1) = .5) and others ( $\alpha$(j,1 = 1.).

As for information network, only the commune of Rouen was given a more fa-

vorable situation ( $\alpha$(j,2) = .1) than the others ( $\alpha$(j,2) = 1.).

## 2 - First simulation : american-type evolution

In order to specify how the Rouen's evolution differed from some ame-

rican-like theoretical cases studied by Allen et al. [2], we made a simula-

tion using the same set of parameters (values in table 2). Results are

plotted on figure 1 for the commune of Rouen (Center), all other communes

being aggregated (Periphery), with dashed lines representing observed evo-

lutions, for each of the six variables. These graphics show the following

tendencies :

- a very rapid decline of industrial employment in the agglomeration center

  and a stagnation in periphery (in fact, industry stagnated between 1954

  and 1975 in Rouen and growed slowly at the periphery, a little more in

  a few communes situated along the river) ;

- a slight move of concentration of exporting tertiary in the center (on

  the contrary, Rouen lost employments mainly to the benefit of two or

  three communes situated on the right bank) ;

- a very large growth of the number of local tertiary employments, which

  are developing quicker in the periphery than in the center (this is al-

  most the observed evolution) ;

Figure 1 : ROUEN calibrage à l'américaine

Figure 1 (suite) : ROUEN    calibrage à l'américaine

COLS BLANCS

OUVRIERS

nombre de résidents

- after a quick departure from the center, regional tertiary employment

  concentrates again on that commune, while stagnating at the periphery

  (the observed evolution carries on the first tendency) ;

- blue collars are concentrating heavily in the center of the agglomeration

  (the observed evolution is almost the opposite) ;

- white collars leave completely the center and disperse in the whole peri-

  phery, more and more farthest (actually, the number of white collars was

  growing faster in the periphery but not leaving the centre of the agglo-

  meration).

    This first trial shows strong differencies between simulated and ob-

served evolution. In order to fit results to observed data, we had to chan-

ge the values of the parameters. It was done according to three principles:

- some of the parameters could be estimated, directly from the data being

  considered : induction rates $\beta$ (number of local and regional tertiary

  employment per inhabitant), external demand D (total number of employment

  of this activity sector at the end of the period), growing rates $\epsilon$ and

  $\eta$ for every employment and resident category.

- other parameters could be induced from other data : the proportion of

  blue collars and white collars working in each of the four economic ac-

  tivities was determined by applying estimations established for french

  urban population ; sensitivity to crowding $\tau$ and $\nu$ were estimated from

  data about space needed by economic activities on the one hand and mean

  density by housing-type in Rouen agglomeration on the other hand.

- the remaining parameters had to be estimated, following the relationships

  between their value and their effects on urban configuration, as esta-

  blished by Allen et al. [2].

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## 3 - Calibration

We started calibration in a very empirical way, changing one value of parameter after another and comparing results obtained by simulation with observed values for each of the six variables in every commune at 1975.

These "empirical" simulations did not yet lead to a good adjustment. We learned however much through them about sensitivity of the model to different values of parameters. The main issues are the following :

- different sets of values for parameters can generate the same type of spatial structure, for example an almost complete decentralization of industry reported upon one or more peripheral communes, or a highly concentrated distribution of industry or exporting tertiary. These kinds of bifurcations are among the most frequently encountered issues of the model. They are mostly determined by respective values of the $\rho$ parameters (table 3).

- Resulting evolution obviously depends on choosen time-lag for simulation. A long period of time furthers apparition of important bifurcations, whereas a shorter period allows a better simulation of smooth increase or decrease as observed in the Rouen commune for industry and exporting tertiary. Sensitivity of the model to parameters values also changes then : for example, variation in the $\rho$ parameters is less determinant upon spatial repartition when time period is short. The model is then more sensitive to $\epsilon$ and $\eta$ parameters.

- from one simulation to another, evolution of spatial repartitions or variation of total quantities of employments and residents are not always directed to the outcome that one would have intuitively expected, by trusting in general significance given to parameters. These counter-intuitive

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Table 3 : Values of parameters and resulting configuration for exporting activities.

| Values for fixed parameters : | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| b | .5 | .3 | | $\emptyset$ | .1 | .005 | .25 | .05 |
| $\beta$ | .24 | .24 | .32 .32 | $\mu$ | 1. | 1. | 1. | 1. |
| Ci | 2. | 2. | | $\nu$ | 1000. | 800. | | |
| Co | 10 | 10 | 8 8 | $\sigma$ | .2 | .2 | | |
| $\epsilon$ | .14 | .12 | .16 .10 | $\tau$ | 2000. | 3000. | 1000. | 2000. |
| $\eta$ | .13 | .15 | | z | .75 .25 .1 .9 .3 .7 .3 .7 | | | |
| D | 503 | 204 | | $\rho$ (3) = $\rho$ (4) = .0015 | | | | |

| Values for : | | Resulting configuration for : | |
|---|---|---|---|
| $\rho$ (1) | $\rho$ (2) | industry | exporting tertiary |
| .02 | .02 | concentrated in the centre | concentrated |
| .015 | .015 | concentrated | dispersed |
| .014 | .014 | dispersed (located on 3 peripheral zones) | concentrated |
| .01 | .01 | dispersed (located on 1 peripheral zone) | concentrated |

results explain themselves on the whole by non-linearities of the system, but make rather difficult a progressive "manual" calibration, because of the number of iterative operations that they imply in choosing values for parameters by trial and error.

That is why, after more than a hundred of these tentative simulations, we decided to use an automatic technique for calibration.

To fit the model's parameter to observed data, we used a non-linear least square method. The actual computation was performed with the help of a library subroutine (called MINUIT) designed by nuclear physicists. It is a flexible minimization program incorporating three different minimization procedures which are used successively and in combination. It may in principle handle as many as forty parameters. It appears however preferable for an efficient convergence to subdivide the parameter set into smaller subsets (of four or five parameters each) to be treated successively.

The adjustments that we obtained until now are not good enough to be considered as a "best fit" and to allow calculation of residuals. But we have a serie of interesting results with various configurations according to different values of parameters. In a number of cases, we obtained a good estimation of the total number of employments and residents for the whole agglomeration but a bad repartition between communes of the center and of the periphery ; in some other cases we obtained an almost perfect adjustement for the center but all variables for the other communes were underestimated. As the model is very sensitive to parameter values, the minimization program can give very different results, depending upon the initial set of values choosen and of the variations which are allowed to them. It

is therefore necessary to examine in detail each result and to modify the initial configuration of parameters values according to their significance and effect on variables evolution. As these effects are not always the same, depending upon the total configuration being considered, the adjustment procedure can take rather a long time. However, it seems to be consistent from one urban case to another, as preliminary results with data for urban agglomerations of Nantes and Bordeaux already showned.

For each of those agglomerations, main points of disagreement between predicted and actual urban evolution concern rates of variation of employment in exporting activities and their spatial repartition. Two further directions for experiment can then be suggested :

- by replacing final potential demand D by a progressive allocation of newly offered employments, less brutal variations could be generated ;
- by modifying accessibility indices of communes, final observed repartition could be better estimated and explanation for these differencies researched afterwards.

We think very plausible to improve the global quality of adjustment between observed and calculated evolution by choosing first a better set of parameters and using the calibration procedure. That is why we do not propose for the moment a final judgment about the assumed too high complexity of the model ( as in Wilson[5]), or about its ability to reproduce observed evolutions. One problem will nevertheless arise when we will find - if we do so - a "satisfying" fit : is the corresponding set of parameters actually the better one ? and is it unique ? Answers to these questions are, of course, critical if we want to use the INTRA URBAN model as a tool for com-

paring evolutions of various urban agglomerations - as we hoped by starting

this study. It is possible that more insights in theoretical issues of the

model are necessary before trying applications - but on  the reverse, only

applications can reveal practical utility of the model. What is more urgent-

ly needed is a serious improvement in the general methodology required for

applications of such complex  dynamic models to real-world situations.


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