Adjustment Dynamics in a Static Input-Output Model

Ernst W. Diehl
Massachusetts Institute of Technology
System Dynamics Group

ABSTRACT

Traditional economic theory emphasizes the determination and characterization of static equilibrium. In contrast, understanding of economic behavior can be enhanced through the use of models that explicitly take into consideration the underlying physical and decisionmaking structure of the system and that allow for disequilibrium. This paper presents an example of such a model. A typical static, open input-output model is translated into an equivalent disequilibrium model. It is shown that objective individual decisions can lead to unintended oscillatory modes of behavior of the overall system. An assumption of perfect information can prevent such undesired oscillations.

The paper also demonstrates a way of communicating system dynamics thinking to an economics audience. The model is developed in progressive steps, a procedure that is widely found in the economic literature. First, stocks are added to a model that originally considers flows alone. Each of three succeeding model changes is then motivated by the results of the previous model and presented as a logical next step towards a more consistent theory. Thus, it is not only the result of the final model as such that is of interest, but also the way the model is developed. Model development is presented as a learning and communication process.

1. MODEL PURPOSE

This paper is written primarily for an economics audience. However, it will be of particular interest for system dynamicists, as well. The purpose of the paper is to communicate one single point that is of paramount importance in system dynamics models but has received much less attention in the economic literature: the importance of modeling explicitly the system's physical structure.[1]

Instead of making a purely abstract case for the need of an explicit representation of the physical structure, I will demonstrate my point with the help of an example. An input-output model has been selected as such an example for its simplicity. In accordance with the purpose of the paper, all the usual assumptions of input-output analysis, such as fixed linear technology, no limits to capacity, homogeneous price development across the sectors, etc. are strictly preserved. Thus the widely recognized shortcomings due to these assumptions will be inherent in the presented model as well. Details of the input-output model will be described in Section 2.

The results will be developed in four steps. First, I will develop a basic model version. Second, I will enrich this model by introducing time delays. These time considerations are usually neglected in input-output models. Third, I will discuss how the results change if a simple inventory control policy is introduced. Finally, I will explore how the intentions of the individual decisionmaker can conflict with the overall performance of the system and demonstrate the difference between perfect and imperfect information in the system's response to shocks.

The paper is not intended to be a critique of the standard formulation of an input-output model as an equilibrium model. For applications where the determination of equilibrium conditions is of primary interest, equilibrium models are the way to proceed. When insight into the dynamics of a model is important, a disequilibrium formulation is needed. The paper demonstrates a way to bridge the gulf between the two approaches.

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[1]In the interest of simplicity, many other points that are stressed in the system dynamics literature, such as distinguishing sharply between different units of analysis, making the model robust under extreme conditions, etc. will not get their deserved attention. The reader with a system dynamics background should have this in mind when discovering what seem to be "flawed" formulations and graphical representations.

It should be stressed at the outset that the focus of the paper lies on deriving qualitative results. The nature of the adjustment dynamics is important, not the quantitative properties of a particular adjustment path. The emphasis is therefore on the validity of the structural form of the derived equations. The parameters of the equation system will only be discussed as far as parameter changes can lead to changes in the nature of the adjustment dynamics.

2. MODEL STRUCTURE

In this section, the input-output model will be described in two forms. First, the simple input-output model is briefly presented in its well known standard equilibrium notation. Second, an alternative representation as a disequilibrium model is discussed.

2.1. THE STANDARD INPUT-OUTPUT MODEL

The standard input-output model assumes a multi-sectoral economy with input-output linkages. The production of one unit of output in one sector requires certain inputs from other sectors. The exact relationship between the inputs required to produce one unit of output is determined by the input coefficients, which are assumed to be fixed. Given knowledge of all input coefficients, it is possible to find an equilibrium such that for each sector the supply of goods equals demand.

The static, open input-output model assumes that final demand and supply of primary factors such as labor and capital are exogenous. Let "x" denote the vector of production, "A" the matrix of input coefficients and "y" the vector of final demand. In equilibrium all produced goods are either needed as intermediate inputs or distributed to the final consumers:

$$x = A*x + y .$$

Solving the equation for "x" and using "I" to denote the identity matrix, we get the well-known Leontief solution:

$$x = (I-A)^{-1} * y .$$

A good introduction to input-output models is provided by Miernyk (1965). United Nations (1973) offers an overview of the simple model and of more sophisticated versions (see also Leontief (1966)). Hansen (1970) gives a good overview of economic equilibrium models in general and shows how input-output models fit in.[2]

The model is commonly used to answer questions such as: "What happens to production if final demand changes?" However, the model can only give part of the answer: the final equilibrium values for production. The model says nothing about the path that production in each sector takes to equilibrium; nor does the model allow any conclusion about the time it takes to reach equilibrium. For a Keynesian-minded politician, who wants to stimulate the economy by increasing government spending, the answers to those questions are essential. It is not only important to know the final equilibrium state of the economy but also the path that the economy takes before it reaches the equilibrium state, if it ever does. Are there wild oscillations to be expected, or will the adjustment be a smooth one? The ability to address these questions is one of the advantages that the alternative disequilibrium formulation provides.

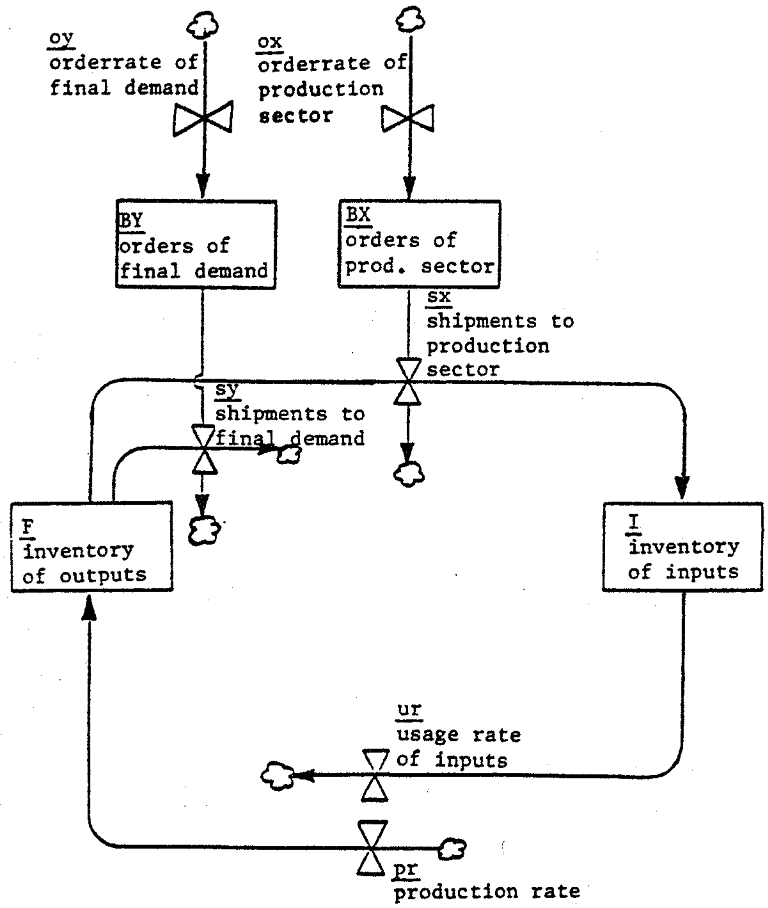
2.2. A DISEQUILIBRIUM FORMULATION

If the transitory path from one point of equilibrium to another is of interest, a model must allow for disequilibrium among its flows. The implication for an input-output analysis is that demand and supply will not be equal at each point in time. One unambiguous way to provide for the possibility of disequilibrium of flows, is to represent explicitly the stocks that intervene between the flows.

Figure 1 provides a graphical description of the model. While it is easy to disaggregate the model, for the sake of simplicity I have chosen to deal with an input-output model in its most aggregated form and to portray a one sector model. The model consists of six flow variables and four intervening. The representation is sufficient to give insight into the production and transport processes that underlie an input-output analysis. Each sector receives

[2]The treatment of dynamics in this paper should not be confused with dynamic input-output models. Dynamic input-output models are still formulated as equilibrium models. They enhance the static model as they allow for growth in capital and production. For more information on dynamic input-output models see the given references above. For an analytical treatment of the underlying assumptions see Dorfman (1958).

Figure 1: Structural diagram of the disequilibrium model



orders from two sources, orders from final demand (oy) and orders from the production sector itself (ox). The production sector ships goods to itself (sx) and to the final demand sector (sy). In a situation of disequilibrium, orders may not equal shipments. The backlogs (BX) and (BY) accumulate the difference between orders and shipments and indicate how many orders have not yet been filled. Each sector holds two kinds of inventories, an inventory of inputs (I) and an inventory of products (F). The inventory for finished goods decouples production from shipments. Similarly, the inventory of intermediate inputs decouples received shipments from the usage rate.

In the equations below uppercase letters indicate stock variables, lowercase letters indicate flow variables or exogenous parameters. Definitions are provided in Appendix II.

Using D() to denote the differential operator, the model so far can be described as:

$$D(BX) = ox - sx$$
$$D(BY) = oy - sy$$
$$D(F) = pr - sx - sy$$
$$D(I) = sx - ur$$

All of these equations are merely definitions. To make the step from a definition to an analysis, behavioral equations are needed. These behavioral equations will be discussed in four subsequent revisions in the next section.

3. MODEL ANALYSIS

One of the advantages of a simulation model is that it is easy to determine the consequences of various changes in the exogenous parameters. For example, one can easily observe the system's reaction to different patterns in final demand. The analysis in this paper concentrates on only one test to facilitate comparisons between the model versions. It is assumed that final demand (oy) is increased permanently by 10% at time 10. The model is formulated in continuous time; a numerical integration procedure is used to simulate the model.

3.1. THE BASE MODEL

The base model does not alter any assumptions of the equilibrium model. Production (pr) and shipments (sx,sy) equal orders (ox,oy) in each instant of time. The production sector's own order rate for input materials (ox) is equal to the usage rate of inputs (ur). The usage rate of inputs is specified by the central input-output assumption as proportional to production rate, with (a) as input coefficient. For the simulation tests, (a) is set to 0.6.
The following equations together with the stock definitions given in Section 2.2. specify the base model. A listing of the model is provided in Appendix Ia.

$$pr = ox + oy$$
$$sx = ox$$
$$sy = oy$$
$$ox = ur = pr*a$$
$$oy = \text{(exogenous)}$$

Solving this equation system for pr yields

$$pr = (1-a)^{-1} * oy \quad .$$

This result is the Leontief solution for a one-sector model (see Section 2.1).

Figure 2: Plot of base case: production

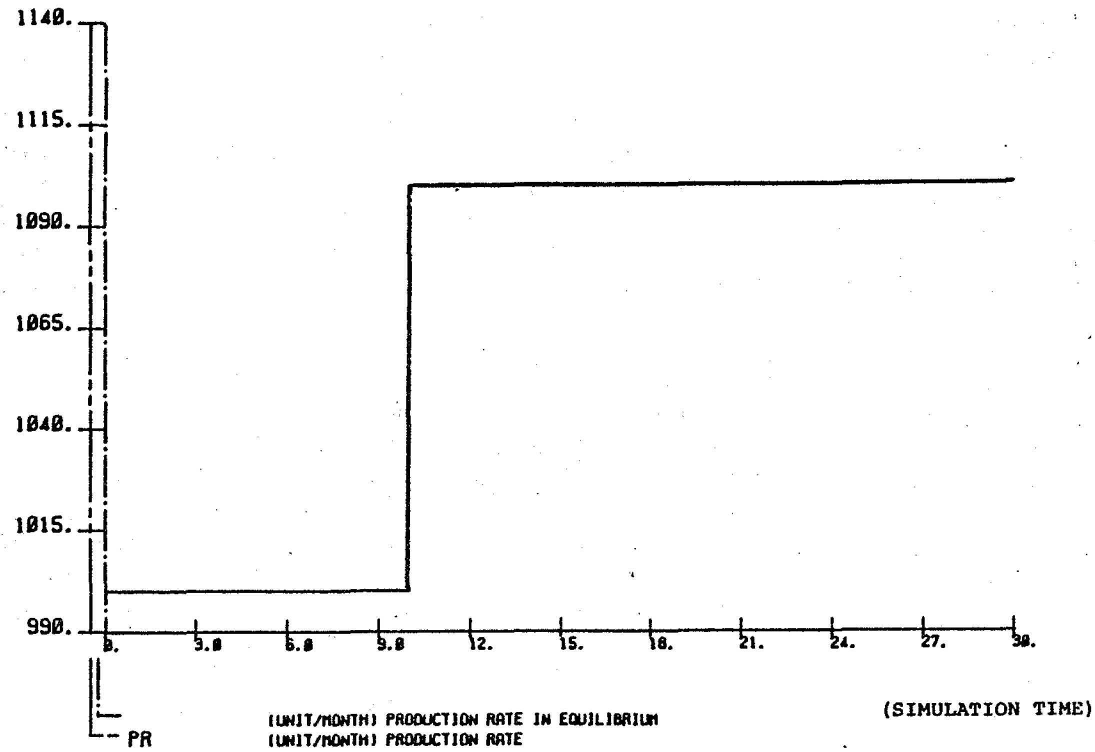


Figure 2 shows the results. Production perfectly traces the step in orders and equivalence of production and orders is preserved in each instant of time. Since no discrepancy between the flows occurs, none of the four stocks changes. The model still assumes instantaneous adjustment, an assumption that does not meet the reality of the goods market. What is needed to come to a better understanding of the dynamics is a more plausible assumption about adjustment times. This is the motivation for the first model revision, described in the following section.

3.2. ADDING TIME DELAYS

Portraying time delays will allow one to come to conclusions about the adjustment path and the time it takes to reach equilibrium. In this model four major time delays can be identified.

Two of these delays are caused by physical constraints. First, goods cannot arrive at their destination at the same time as they leave the factory. Therefore, the equation equating the arrival and shipment rates in the base case has to be substituted by an equation that takes the shipping delay into account. A third-order delay function, with an average lag of shipment time (ts), is used as an approximation for this delay. For a description of the dynamic properties of a third-order delay process and its ability to approximate delays of transportation processes see (Forrester, 1961 - Appendix H). Second, production requires time. The production finishing rate (fr) is a delayed function of the production initiation rate. Again a third-order delay is used as an approximation with an average lag of production time (tp).

The third major time delay is a perception delay. It takes time to perceive long-lasting changes in the incoming order rate and to plan production accordingly. There is a lot of noise in a real economic system and a manager is more likely to tolerate fluctuations in the order backlog in the production rate The indicated production rate (P) is formulated as adaptive to orders, so that the noise is smoothed out. The time to perceive long-lasting changes in the order rate (tc) is a weighting factor that specifies, how fast new information about order changes influences expectations about the appropriate level of production.

One further time delay has to be considered. The base case assumed that orders would be filled in the very second they were placed. In a model with real time assumptions, one has to model the existing delivery delay (td). Processing an order takes time. The delivery delay represents the time between placing an order and the time the product leaves the factory. The changes can be summarized as:

Model I	Modifications
D(I) = sx-ur	D(I) = ar-ur
	ar = delay3(sx,ts)
D(F) = pr-sx-sy	D(F) = fr-sx-sy
	fr = delay3(pr,tp)
pr = ox+oy	pr = P
	D(P) = (ox+oy-P)/tc
sx = ox	sx = BX/td
sy = oy	sy = BY/td

It should be noted that these equations are not formulated in a robust way. For example, shipments are not constrained by the number of products hold in inventory. The model formulation does not prevent inventory from becoming negative. Thus, one should think of this model as a linearization of a more complicated non-linear model about a "normal" operating point.

The complete equations for the revised model are given in Appendix Ia. Appendix Ib lists the parameter values used for the simulations. The adjustment dynamics are shown in Figure 3. Adding time delays changes the pattern of the adjustment process. Production does not jump instantaneously to the new equilibrium value, but approaches it smoothly. The larger the values for the time delays, the longer it takes to reach equilibrium. The shape of the adjustment path, however, is independent of the parameter values. But is the adjustment process shown actually based on plausible behavior? Figure 4 shows how the inventories react. Both inventories decrease rapidly after the order increase, and level off later as they reach a new, lower equilibrium value. This result is not realistic. Assuming that inventories are held as a buffer against uncertainty in demand, production and deliveries of inputs, one would expect that inventories are kept in a certain proportion to the level of production.[3] Given the situation of a long-lasting production increase, inventories should reach a higher equilibrium value, not a lower one.

What is the reason for the unrealistic behavior of the model? Starting with the initial increase in orders, products get shipped out of finished goods inventory while production slowly adjusts to the higher level of orders. Shipments respond faster to changes in orders than production does. Thus, the inventory of finished goods is reduced. As for input inventories, the inputs needed are used immediately as production gets underway. In response to the increase of production more orders for inputs are issued. However, it takes time for those inputs to arrive at the factory. Thus, input inventories decrease as well.

So far, the decisionmakers in the system do not control inventories. Thus, when the adjustment process finally finishes, there is no force to bring inventories up. Inventories in the system would follow a random walk in response to variations in final demand. If the step in final demand had been higher, this policy could have been disastrous, because inventories could have run out entirely.

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[3]See Section 3.3. for a discussion of inventory holding motives.

Figure 3: Time delays added: production

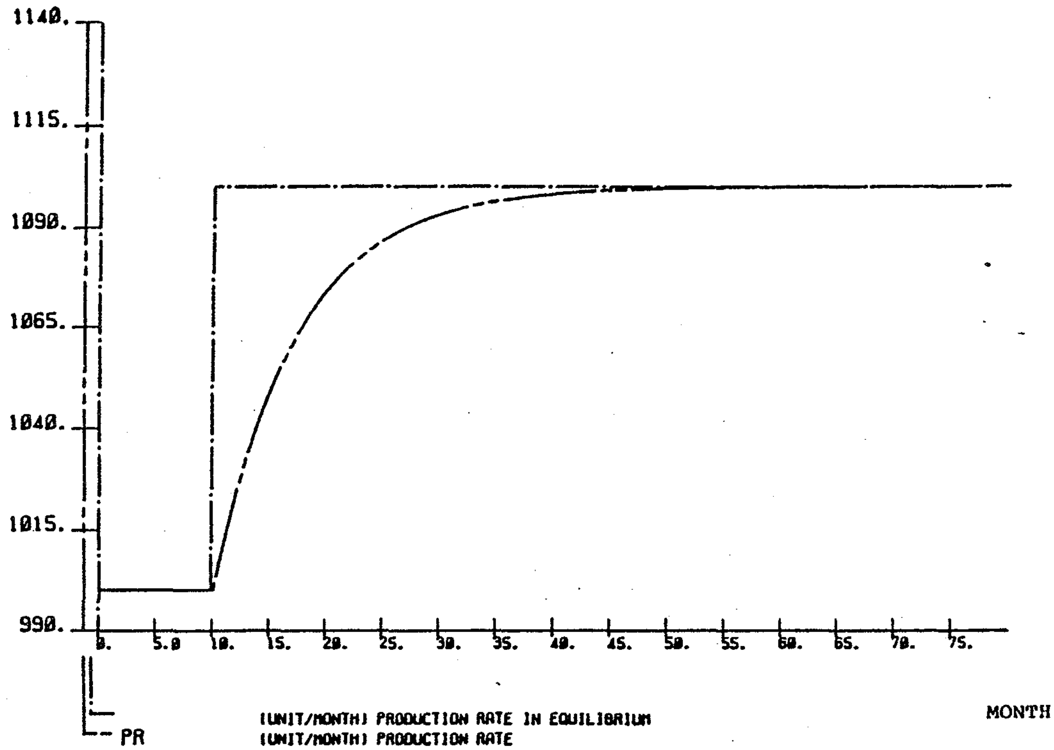
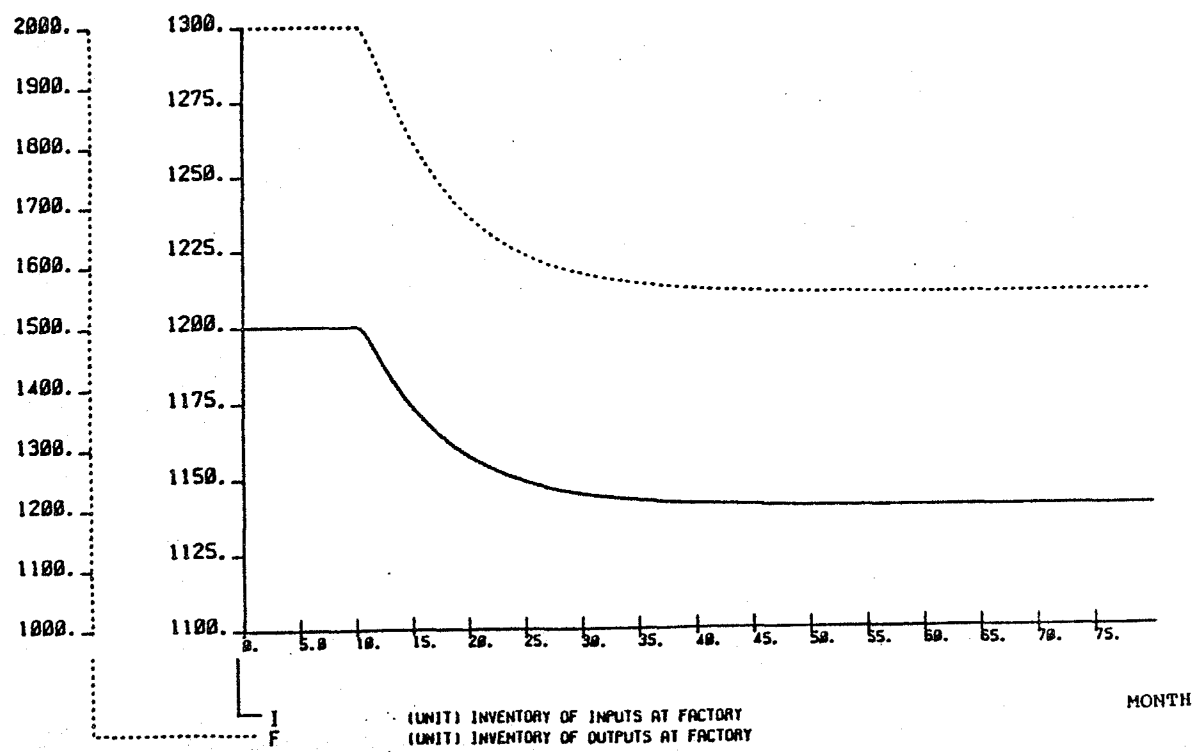


Figure 4: Time delays added: inventories



Thus, inventory control has to be added to the model. This is the motivation for the revisions made in the next section.

3.3. ADDING INVENTORY CONTROL

A variety of possible inventory holding motives have been discussed in the literature (Blinder, 1980). The production-smoothing model of inventory behavior is probably the most widely accepted (Holt, 1960). For a critical discussion of this assumption see Blinder (1984). Following the results of the literature, inventory is assumed to be proportional to expected demand. A constant number of months' inventory is assumed to be held to cover the production delay.

While there is a certain agreement that the production-smoothing model applies for inventories of finished goods, much more uncertainty surrounds the motives of holding inventories of inputs (Blinder, 1980). But for a model with fixed prices, where no speculative behavior is possible, it seems justified to apply the same production smoothing model to the inventory of inputs as well. To avoid disruptions of the production schedule, inputs must be readily available. The higher the level of activity, the higher the input inventory has to be.

Due to the similar assumptions made about the holding of input and finished goods inventories, the model formulations for adding inventory control are modeled in a parallel fashion. The order rate is expanded to allow for orders due to input inventory adjustment; the production rate is expanded to allow for orders due to finished product inventory adjustment. Inventory adjustment is accomplished by comparing actual inventories with desired inventories. The adjustment parameters (tf) and (ti) represent the speed with which management strives to correct the difference. The desired inventory coverage is given by (cf) and (ci). The equations below show the model modifications.

Model II	Modifications
ox = P*a	ox = P*a+(id-IE)/ti
	D(IE) = ox-ur
	id = (ci+ts+td)*P*a
pr = P	pr = P+(fd-FE)/tf
	D(FE) = pr-sx-sy
	fd = (cf+tp)*P

For a complete model and the values of the chosen parameters values for ci, cf, ti, and tf see Appendix Ia and Ib. Inventories are initialized in equilibrium.

Note that the inventory control is not based simply on the inventories at factory (I) and (F), but on an extended inventory definition, (IE) and (FE). A decisionmaker does not only have to consider the inventory at the factory but also the units already in process. Thus, the extended inventory of finished products (FE) includes the work in progress and integrates the difference between production initiation rate and shipments. In an analogous way, the extended inventory for intermediate inputs (IE) includes the issued but not-yet-filled orders and integrates the difference between order rate and usage rate. Desired inventories (id,fd) are similar extended and include desired inventory at the factory and desired units in process.

Figures 5 and 6 show the result of the revised model. The results are rather surprising. Instead of a smooth adjustment oscillations in production and inventories occur. All the behavioral assumptions in the model are in the individual firm's best interest. However, the behavior of the system as a whole turns out to be undesirable.[4] Why?

Before answering this question, the sensitivity of the results to the time parameters has to be discussed. While the results of version II in qualitative terms are unaffected by any changes in parameter values, the same is not true for extreme assumptions about time parameters in the current version. For a combination of small inventory coverage and large inventory adjustment times, the oscillatory mode might disappear. These results and a more detailed sensitivity analysis can be derived by showing that for a certain range of parameter values, all eigenvalues of the model only have real parts. A technical paper on the model is currently being prepared.

What causes the fluctuations in the adjustment process? The underlying cause is an information problem. There is no way for the firms in the production sector to tell whether the incoming orders are issued "merely" because of inventory adjustments in other companies or whether the incoming orders reflect long-lasting changes in demand. The assumption of incomplete information seems to be realistic. At least, it is more realistic than to assume that each company has perfect knowledge of what other companies are doing, why they are doing it, and how long they intend to keep doing it.

If this explanation is true, then a model with perfect

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[4]This value judgment is based on an implicit assumption about a cost function. Wide swings in production can be very costly, as well as difficult to manage.

Figure 5: Inventory control added: production

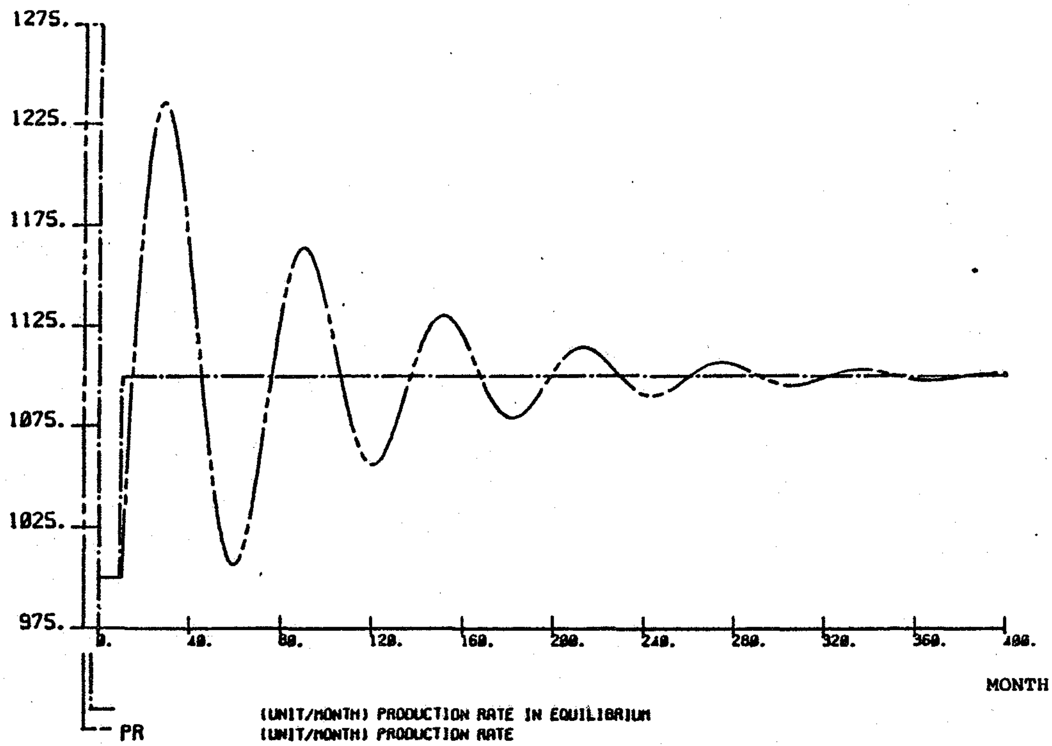
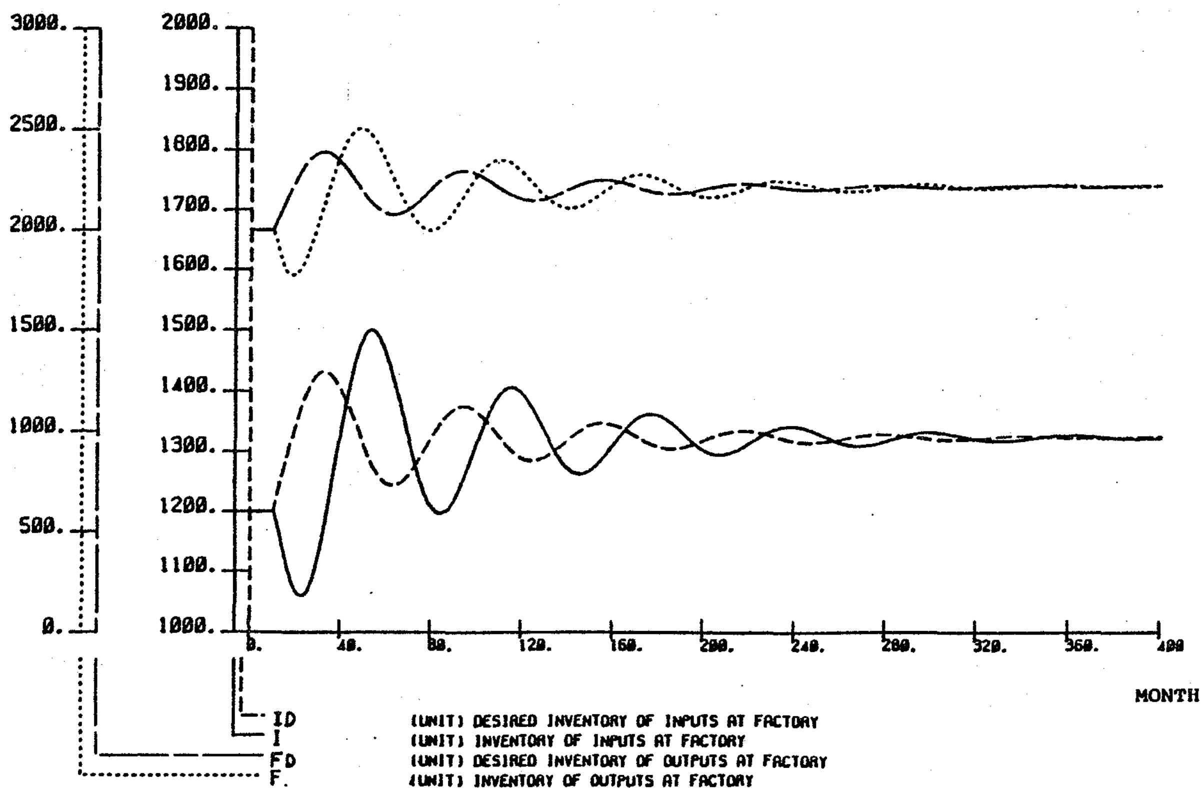


Figure 6: Inventory control added: inventories



information should not show any such oscillations. The next section will show how the model can be enhanced to test this hypothesis.

3.4. ADDING PERFECT INFORMATION

In order to test the dynamic hypothesis that incomplete information is the cause of fluctuations, the model has to be enhanced in such a way that the situations of complete and incomplete information can be compared to each other. Complete information in this context means that each issued order is marked with a label that tells whether the order is issued because of inventory adjustment or whether it is ordered as part of the long-lasting production plan.[5]

If the firms had this knowledge, they would be in a better situation to determine their inventory goals. Changes in orders and production for inventory adjustment are only temporary. Thus, a decision-maker would not make the desired inventory a function of these purely temporary signals.

The same is true for decision about the amount to produce and the amount of inputs to order. Reluctant to put unnecessary fluctuations into the production and order lines, a plant manager would in the case of perfect information not consider temporary signals as inputs for the production decision. It is the natural function of the inventories (not of the order or production line) to be a buffer for exactly these cases.

The implementation of these ideas in the model is rather simple. All orders and products are labeled with regard to their original causes (long-lasting demand or temporary inventory adjustment). To do so most of the equations are split into two parts. A complete listing of the new model version is given in Appendix Ia.

Note from this listing that only one time parameter has been changed - the time to perceive changes in order rate (tc) is now split into two parameters: (tc1) and (tc2). This distinction makes it possible for the modeled sector to adjust to changes in long-lasting orders in a different manner than to changes in orders issued due to inventory adjustment reasons. However, as long as (tc1) equals (tc2) nothing has changed. In this case the refined model necessarily produces the same results as the previous one; it expresses the same logic.

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[5]It is still assumed that firms do not have perfect foresight. Decisions are still formed adaptive to orders.

Figure 7: Perfect information: production

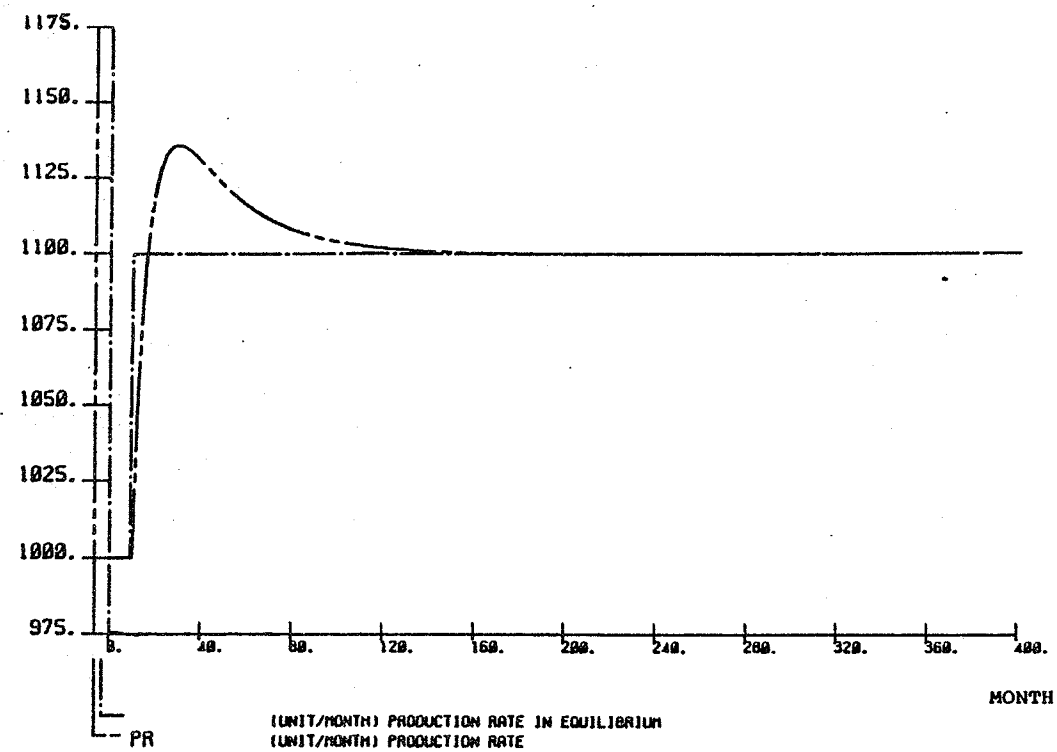
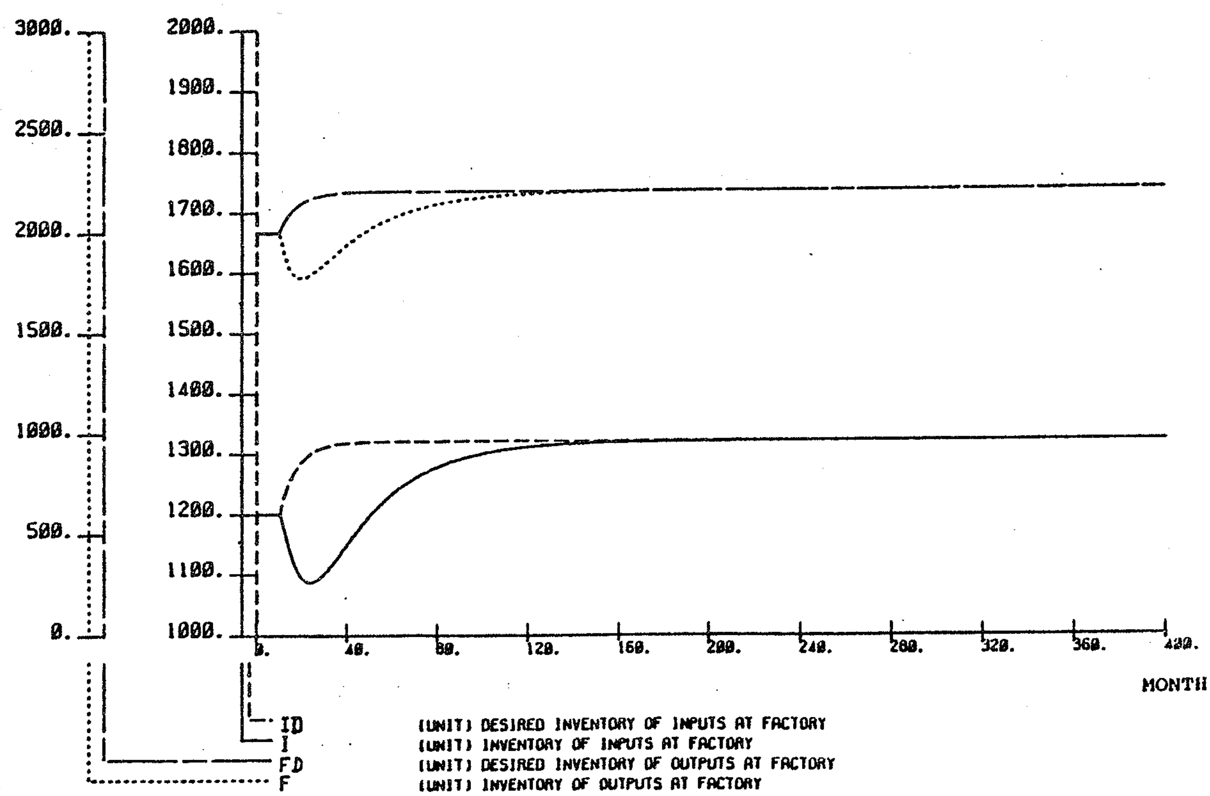


Figure 8: Perfect information: inventories



However, the model now allows one to test for the case of perfect information. The parameter (tc2) is set in the limit to infinity. This single change deactivates all merely temporary signals: Orders due to inventory adjustment no longer impact the production and inventory holding decisions.

The results of the run with perfect information are shown in Figures 7 and 8. The desired inventories now smoothly approach the desired equilibrium. Inventories in the beginning fall and then adjust smoothly to their equilibrium value without oscillations. The behavior of the rate of production might seem surprising at first, since it overshoots its long-run equilibrium once and then smoothly approaches it. A careful examination shows, however, that the overshoot is a necessary part of any adjustment path. Given the fact that inventories have to fall in the beginning due to the physical and perception delays in the system, production has to make up later on for insufficient long-run inventories. Production necessarily has to overshoot its long-run equilibrium once in order to establish the desired inventory values for that equilibrium.

An assumption of complete information is thus sufficient to lead to a non-oscillating adjustment path of the system. As argued earlier, such a perfect situation is unfortunately not realistic.

4. CONCLUSIONS

A way to achieve insight into the implicit dynamics of an economic equilibrium model has been demonstrated by translating the equilibrium model into a disequilibrium form and making the underlying physical structure explicit. A static, open input-output model has been used as an example. The results can be summarized in four points:

- Equilibrium formulations are justified for cases where the adjustment dynamics are smooth and fast or where the dynamics are of no interest.

- A disequilibrium model requires the introduction of behavioral assumptions for the control of stocks, which intervene between the model's flows.

- Even if the stock control is formulated in a way that best suits the individual's objectives, the overall performance of the system can be undesirable.

- The major reason for the discrepancy between subjective intentions and objective performance is an information problem. The potential for the discrepancy can be avoided if complete information about the whole system is available instantaneously to all individual decision units.

The approach can be applied to more sophisticated versions of inpt-output models and to economic equilibrium models in general to illustrate the implicit adjustment dynamics of such models. Current economic problems suggest that a dynamic approach might be more valuable in policy design.

REFERENCES

Blinder, Alan S. Inventories in the Keynesian macro model. NBER working paper 460, NBER, February 1980.

Blinder, Alan S. Can the production smoothing model of inventory behavior be saved?. NBER working paper 1257, NBER, January 1984.

Dorfman, Robert; Samuelson, Paul A. and Solow, Robert M. Linear programming and economic analysis. New York:McGraw Hill, 1958.

Forrester, Jay W. Industrial dynamics. Cambridge, Mass.:The M.I.T. Press, 1961.

Forrester, Nathan. A dynamic synthesis of basic macroeconomic theory: implications for stabilization policy analysis. PhD thesis, Massachusetts Institute of Technology, August, 1982.

Hansen, Bent. A survey of general equilibrium systems. New York:McGraw Hill, 1970.

Holt, Charles C.; Modigliani, Franco; Muth, John F. and Simon, Herbert A. Planning production, inventories and work force. Englewood Clifs, N.J.:Prentice Hall, 1960.

Leontief, Wassily W. Input-output economics. New York:Oxford University Press, 1966.

Miernyk, W.H. The elements of input-output analysis. New York:Random House, 1965.

United Nations. Input-output tables and analysis. Series F No.14, Rev. 1, New York, 1973.

APPENDIX Ia.: MODEL DOCUMENTATION

I.: Base Model

```
D(I)=sx-ur
*
ur=pr*a
ox=pr*a
*
*
*
*
*
*
-------------------
D(F)=pr-sx-sy
*
pr=ox+oy
*
*
*
*
*
*
*
-------------------
D(BX)=ox-sx
*
*
D(BY)=oy-sy
sx=ox
*
*
sy=oy
```

II.: Time Delays

```
D(I)=ar-ur
ar=delay3(sx,ts)
ur=pr*a
ox=P*a
*
*
*
*
*
*
-------------------
D(F)=fr-sx-sy
fr=delay3(pr,tp)
pr=P
D(P)=(ox+oy-P)/tc
*
*
*
*
*
*
-------------------
D(BX)=ox-sx
*
*
D(BY)=oy-sy
sx=BX/td
*
*
sy=BY/td
```

III.: Inv. Control

```
D(I)=ar-ur
ar=delay3(sx,ts)
ur=pr*a
ox=P*a+(id-IE)/ti
*
*
id=(ci+ts+td)*P*a
*
*
D(IE)=ox-ur
-------------------
D(F)=fr-sx-sy
fr=delay3(pr,tp)
pr=P+(fd-FE)/tf
D(P)=(ox+oy-P)/tc
*
*
fd=(cf+tp)*P
*
*
D(FE)=px-sx-sy
-------------------
D(BX)=ox-sx
*
*
D(BY)=oy-sy
sx=BX/td
*
*
sy=BY/td
```

IV.: Perf. Inform.

```
D(I)=ar-usr
ar=delay3(sx,ts)
ur=pr*a
ox=ox1+ox2
ox1=P1*a
ox2=P2*a+(id-IE)/ti
id=id1+id2
id1=(ci+ts+td)*P1*a
id2=(ci+ts+td)*P2*a
D(IE)=ox-ur
-------------------
D(F)=fr-sx-sy
fr=delay3(pr,tp)
pr=P+(fd-FE)/tf
P=P1+P2
D(P1)=(ox1+oy-P1)/tc1
D(P2)=(ox2   -P2)/tc2
fd=fd1+fd2
fd1=(cf+tp)*P1
fd2=(ci+tp)*P2
D(FE)=pr-sx-sy
-------------------
BX=BX1+BX2
D(BX1)=ox1-sx1
D(BX2)=ox2-sx2
D(BY)=oy-sy
sx=sx1+sx2
sx1=BX1/td
sx2=BX2/td
sy=BY/td
```

APPENDIX Ib.:

EXOGENOUS VARIABLES, INITIAL VALUES

```
oy=400+test
test=step(40,st)
st=10
a=0.6
cf=2
ci=2
tc=3
tc1=3
tc2=1E7
td=0.5
tf=6
ti=6
tp=2
ts=0.5
-------------------
F=2000
FE=4000
I=1200
IE=1800
P=1000
P1=1000
P2=0
BX=300
BX1=300
BX2=0
BY=200
```

APPENDIX II.: LISTING OF VARIABLES

__Endogenous variables:__

```
BX =(unit) backlog of orders from production sector
BX1=(unit) backlog of orders from p.s. (long-lasting)
BX2=(unit) backlog of orders from p.s. (temporary)
BY =(unit) backlog of orders from final demand sector
F  =(unit) inventory of outputs
FE =(unit) extended actual inventory of outputs
I  =(unit) inventory of inputs
IE =(unit) extended actual inventory of inputs
P  =(unit/month) indicated production rate
P1 =(unit/month) indicated production rate (long-lasting)
P2 =(unit/month) indicated production rate (temporary)
ar =(unit/month) arrival rate of inputs
fd =(unit) desired inventory of outputs
fd1=(unit) desired inventory of outputs (long-lasting)
fd2=(unit) desired inventory of outputs (temporary)
fr =(unit/month) production finishing rate
id =(unit) desired inventory of inputs
id1=(unit) desired inventory of inputs (long-lasting)
id2=(unit) desired inventory of inputs (temporary)
ox =(unit/month) production sector's order rate on inputs
ox1=(unit/month) p.s. order rate on inputs (long-lasting)
ox2=(unit/month) p.s. order rate on inputs (temporary)
oy =(unit/month) final demand's order rate
pr =(unit/month) production rate
sx =(unit/month) shipments to production sector
sx1=(unit/month) shipments to p.s. (long-lasting)
sx2=(unit/month) shipments tp p.s. (temporary)
ur =(unit/month) usage rate of inputs
```

__Exogenous variables__

```
a  =(unit/unit) input coefficient
cf =(month) inventory coverage for finished goods
ci =(month) inventory coverage for inputs
st =[step(...,st)] occurrence time of step in orders
tc =(month) time to perceive order changes
tc1=(month) time to perceive order changes (long-lasting)
tc2=(month) time tp perceive order changes (temporary)
td =(month) delivery delay
tf =(month) adjustment time for output inventory
ti =(month) adjustment time for input inventory
tp =(month) production time
ts =(month) shipment time
```