

# Two different methods of embedding the optimization in simulation on model DYNBALANCE(2-2)

Elżbieta Kasperska, Damian Słota

Institute of Mathematics

Silesian University of Technology

Kaszubska 23, Gliwice 44-100, Poland

e-mail: elakaspe@polsl.gliwice.pl, damslota@polsl.gliwice.pl

## Abstract

*The purpose of this paper is to present the two different methods of embedding the optimization in simulation on model DYNBALANCE(2-2). This model follows the versions, named: DYNBALANCE(1-3) and DYNBALANCE(3-1). Now, authors confronts two methods of embedding the optimization in simulation on models type SD. First of the undertaking of the problem refers to the idea of Legras about so called "pseudosolution" of equation:  $Ax - b = 0$  which minimize the norm of these differences. The second formulating of the problem takes advantage of Linear Programming and can be named: "embedding linear programming in System Dynamics".*

**Keywords:** System Dynamics Method, Optimization and Simulation, Model DYNBALANCE(2-2), Model DYNBALANCE(2-2-LP).

## 1 Introduction

The idea of embedding the optimization in simulation on models type System Dynamics authors have undertaken many years ago. First, Kasperska in her paper (Kasperska 1980) has asked a question about identification and construction of decision policies which has "optimization" character. Then Kasperska has been studying the problem of artificial intelligences and was inspired by the works of Keloharju (Keloharju 1977) and mainly by prof. G. Coyle (Coyle 1996, Coyle 1998, Coyle 1999). But, only few years ago Kasperska and her colleagues: Słota and Mateja-Losa, have possibilities to take experiments on models using languages: Professional Dynamo 4.0 and COSMIC and COSMOS. The object of experimentation were models of dynamical balances of production and balances of raw materials, created by Kasperska and named: DYNBALANCE(1-3), DYNBALANCE(3-1). From books and articles of prof. Coyle follows the idea of "simulation during optimization" (see (Coyle 1996, Coyle 1999)). The Kasperska has gone the opposite way. She was interested in embedding the optimization in simulation on System Dynamics

models. But not only. In papers (Kasperska and Słota 2000, Kasperska, Mateja-Losa and Słota 2000b, Kasperska, Mateja-Losa and Słota 2001) Kasperska with her colleagues, applied the idea of Coyle as well. Now, the attention is focus on two different methods of embedding the optimization in simulation on the example of model DYNBALANCE(2-2). In next section they will be presented and further some interesting results will be discussed and some questions of interest will be undertaken.

## 2 Two methods of embedding the optimization in simulation on model DYNBALANCE(2-2)

For presenting the methods of embedding the optimization in simulation on model type System Dynamics, some mathematics is necessary. Now from the experience, authors appreciates the second ("historically invented") method like much valuable but now starts from description of first method signalized in the introduction.

Before the presentation the idea of this method we have to reflect upon the question: what really we want to optimize, and what for? Generally speaking, we model and simulate the dynamics of modelled systems, to: "diagnose the causes of faulty behavior and tune the system feedback loops to get better behavior" (see (Coyle 1996)). So the measurement of that "faulty behavior", seems to be very important issue for the modelers. One of such measure can be the norm (mathematically precisely speaking euclidesian norm) of the defences of matrix:  $Ax - b$ , where the matrixes:  $A$ ,  $x$  and  $b$  have the meaning comprehensible fully after presentation the structure of model DYNBALANCE(2-2). So, its time for necessary details. On Figure 1 the Reader can study the main structure of the model, in Łukaszewicz's symbols (Łukaszewicz 1975, Łukaszewicz 1976, Kasperska 2002a). Below the meanings of matrixes:  $A$ ,  $x$  and  $b$  can be recognize in presented matrix equation  $Ax = b$ :

$$\begin{pmatrix} q11 & 0 & q12 & 0 \\ 0 & q21 & 0 & q22 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ ucp11 \cdot q11 & 0 & ucp21 \cdot q12 & 0 \\ 0 & ucp12 \cdot q21 & 0 & ucp22 \cdot q22 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} rm11 \\ rm12 \\ rm21 \\ rm22 \end{pmatrix} = \begin{pmatrix} frdp1(t) \\ frdp2(t) \\ sourc1(t) \\ sourc2(t) \\ b5 \\ b6 \\ b7 \\ b8 \\ b9 \\ b10 \end{pmatrix} \quad (1)$$

The variables:  $frdp1(t)$  and  $frdp2(t)$  are not presented on Figure 1, but the Reader can recognize them by follow the Appendix 1. The idea of solving the matrix equation (1) uses the method of Legras (Legras 1974) by finding the so called "pseudosolution", which minimize the norm of underdetermined equation (1). This is the euclidesian norm, so it is the square root of sum of squares of discrepancies  $(Ax - b)_i$ , for  $i = 1, 2, \dots, 10$ . So, the found solution is that give the best "fitting" of balance (modelling by equation (1)).

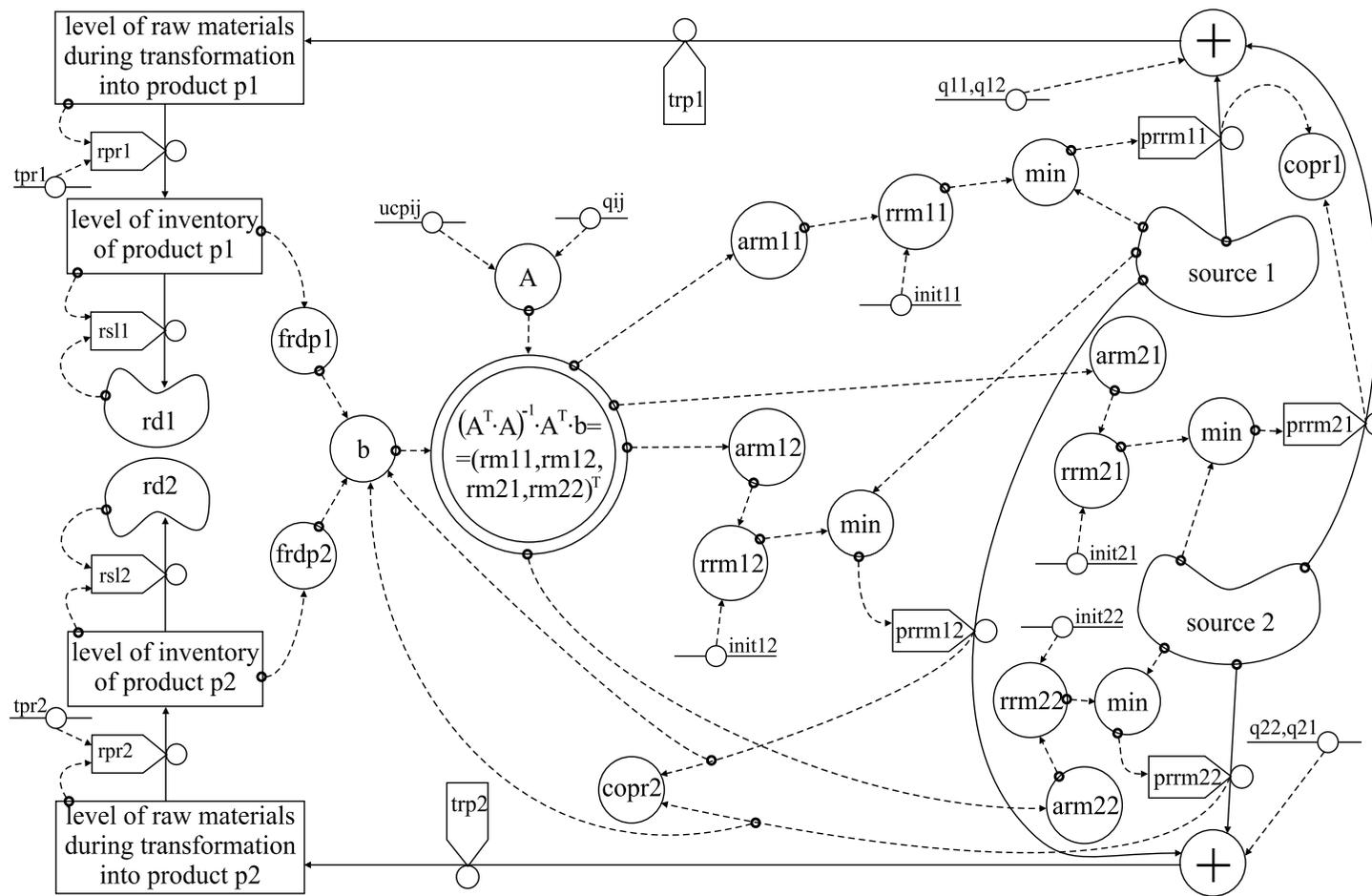


Figure 1. Main structure of the model DYNBALANCE(2-2)

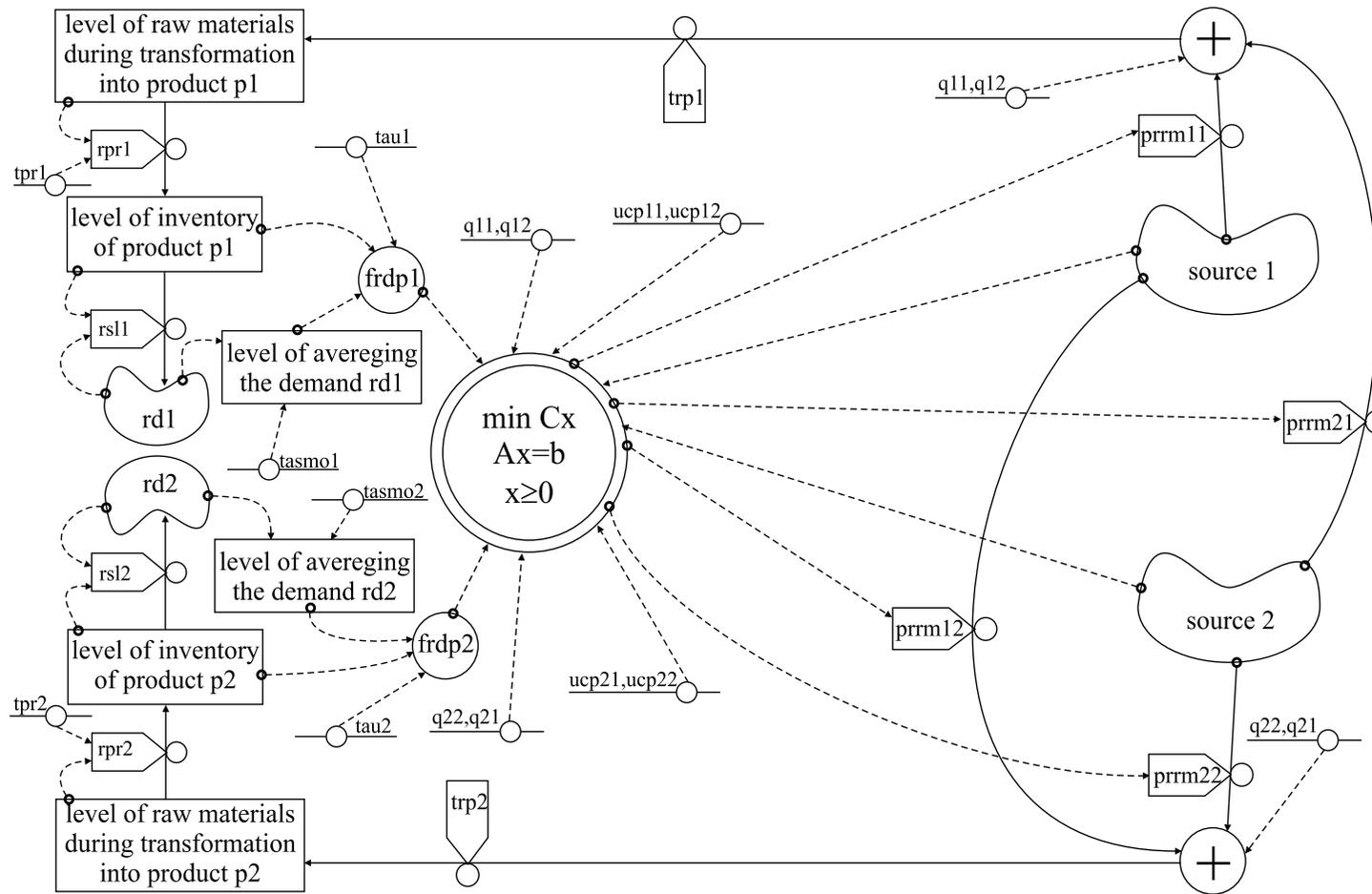


Figure 2. Main structure of the model DYNBALANCE(2-2-LP)

Technically speaking, the solving of matrix equation (1) is the problem of programming specific works on matrixes, in order to attain  $x$ :

$$x = \begin{pmatrix} rm11 \\ rm12 \\ rm21 \\ rm22 \end{pmatrix} = (A^T \cdot A)^{-1} \cdot A^T \cdot b.$$

The formula was examined by authors and for given number values we have gotten results, that will be described in next section.

Now, we start to present the second of promised method of embedding optimization in simulation on System Dynamics models. The idea is completely simple. Locally, in model SD, functions the decision rule, which is optimal, in sense of classical Linear Programming. So, the rule models the decision, which gives the best desired (in sense of accepted criteria) value of optimized flow (here: flows). The combined effect of embedding this LP solver in SD models, is that we can examine the influences of "locally" optimal decision on the dynamics of a whole system.

The formalization of such idea, refered to the model DYNBALANCE(2-2), gives its corrected version: DYNBALANCE(2-2-LP), which is presented on Figure 2.

The symbol in double surrounding represents the classical formulation of the problem of minimization the linear function (in matrix description:  $f(x) = cx$ ) at the constraints given by set of equations:  $Ax = b$  and  $x \geq 0$ . At the conditions of our model DYNBALANCE(2-2-LP), this symbol means detally:

$$\begin{aligned} f(x) = cx &= \sum_{i=1}^8 c_i x_i, \\ Ax &= b, \\ x &\geq 0, \end{aligned}$$

where:

$$c = (ucp11 \cdot q11, ucp21 \cdot q12, ucp12 \cdot q21, ucp22 \cdot q22, 0, 0, 0, 0),$$

$$A = \begin{pmatrix} q11 & q12 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & q21 & q22 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$b = (frdp1(t), frdp2(t), sour1(t), sour2(t)).$$

This matrix formulation come from condition of task of minimalization by constraints, namely:

\*) the condition for size of production, for each product separately:

$$\begin{aligned} q11 x_1(t) + q12 x_2(t) &\geq frdp1(t), \\ q21 x_3(t) + q22 x_4(t) &\geq frdp2(t); \end{aligned}$$

\*\*\*) the constraints of size of supply of raw materials, for each source separately:

$$\begin{aligned} x_1(t) + x_3(t) &\leq sourc1(t), \\ x_2(t) + x_4(t) &\leq sourc2(t); \end{aligned}$$

\*\*\*) of course, the values of rates (of raw materials number one and two for production of products number one and two) can not to be negative.

Before embedding the constructed linear programming solver to the program in Dynamo, we first solve the problem "manually" for the time 0 of simulation. To complete the information, lets determine, in the numerical form the problem of optimization. So:

$$\min f(x) = \min (1.5 x_1 + 2 x_2 + 2 x_3 + x_4 + 0 x_5 + 0 x_6 + 0 x_7 + 0 x_8),$$

where:

$$\begin{cases} x_1 + 2 x_2 - x_5 = 450, \\ x_3 + 2 x_4 - x_6 = 450, \\ x_1 + x_3 + x_7 = 200, \\ x_2 + x_4 + x_8 = 350. \end{cases}$$

We have received the solution:

$$\begin{cases} x_1 = 200, \\ x_2 = 125, \\ x_3 = 0, \\ x_4 = 225, \\ x_5 = 0, \\ x_6 = 0, \\ x_7 = 0, \\ x_8 = 0. \end{cases}$$

For such solution, the minimum of  $f(x)$  is:

$$\min f(x) = 1.5 \cdot 200 + 2 \cdot 125 + 225 = 775.$$

We have the stres that such embedding the Linear Programming in System Dynamics has the dynamical character. For each step of simulation the Linear Programming solver will be calculated the optimum and "embedding" it to the program of System Dynamics. The condition for Linear Programming changed dynamically and in consequences, influence the System Dynamics model as well. So this is the fully feedback! The idea is compatible with base assumption of System Dynamics modelling.

### **3 The results of experiments, with embedding optimization in simulation, on model DYNBALANCE(2-2)**

Lets pay attention now on results of experiments on model DYNBALANCE(2-2). Many interesting experiments were planned by Kasperska and performed by Słota by using Professional Dynamo. The full program of model is located in Appendix 1. On Figures 3–7 the main characteristics of variables of model DYNBALANCE(2-2) are presented. The Reader, who will be interested in details, has possibilities to compare

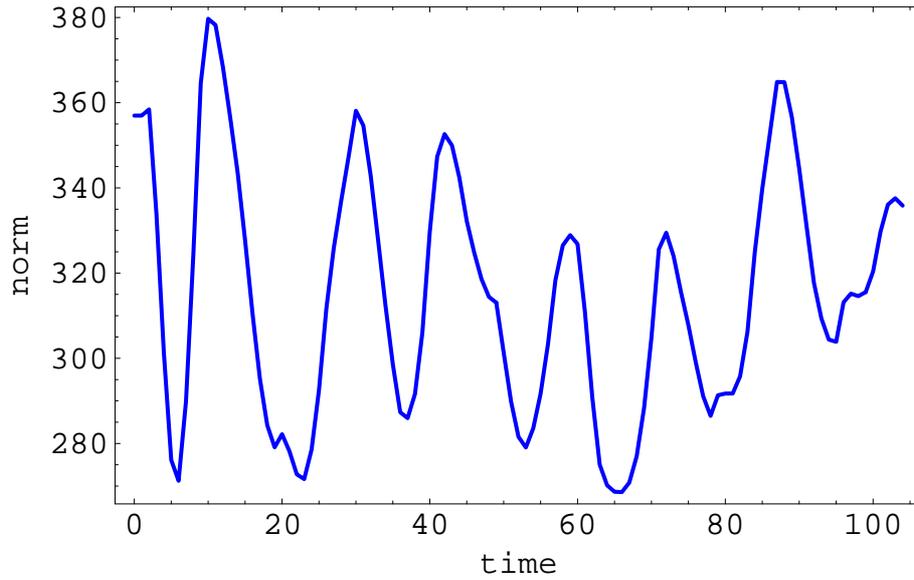


Figure 3. The characteristic of main variable  $norm$  in basic experiments ( $lin1(0) = 700$ ,  $lin2(0) = 700$   $source1 = 200$ ,  $source2 = 350$ )

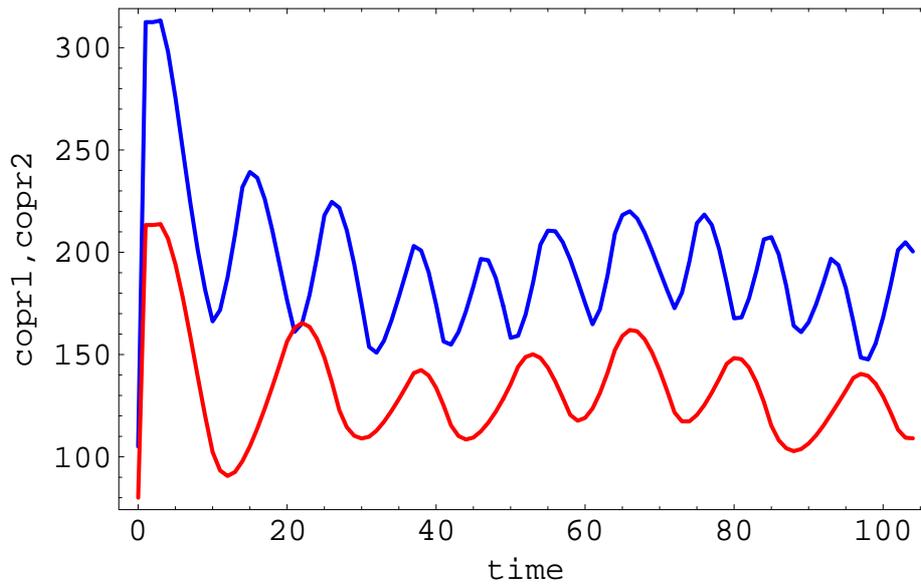


Figure 4. The characteristics of costs of production for products  $P1$  ( $copr1$  – blue line) and  $P2$  ( $copr2$  – red line) in whole horizon of simulation ( $lin1(0) = 700$ ,  $lin2(0) = 700$   $source1 = 200$ ,  $source2 = 350$ )

such important (in sense if "quality" of fitting the balance:  $Ax = b$ , and in sense of dynamics of whole system) variables, like:  $norm$ ,  $copr1$ ,  $copr2$   $lin1$ ,  $lin2$ ,  $rsl1$ ,  $rsl2$   $prof1$ ,  $prof2$ ,  $profit$ . Under the figures, authors have indicated the characteristic values of parameters, which can be change in different experiments.

The "dramatic" influence of parameters:  $lin1(0)$ ,  $lin2(0)$  and the changing of  $source1$  and  $source2$  for some characteristics of system Reader can see on Figures 8–11. Under the condition of constraints of supply from sources of raw materials, the

problem of stopping the production has occurred and in consequences the losse of profit from production. The dynamics of production of product  $P2$  in such conditions is shown on Figure 10. The dynamics of inventory of product  $P2$  is shown on Figure 9.

The dynamics of profits from sale of products  $P1$  and  $P2$  and profit from whole production under the condition of constraints of sources of raw materials Reader can see on Figure 11 (and compare it from Figure 7).

The authors think that for Readers the presentation of detailed table of values of discrepancies  $blb1, blb2, \dots, blb10$  (entering the *norm* – see Appendix 1) will be interesting, too. It is possible to see (and compare) the influence of the components of *norm* on the final value. So, the authors have decided to present detailed results in Table 3.

During creating the model and simulating the dynamics, authors have performed so many experiments, that it is not possible to present them all in this paper. It should be stress that ideas of authors have gone not only in direction of minimizing of cost of production but of course in dual problem: the maximizing of profit. The efforts are not finished. So it's time for the conclusions.

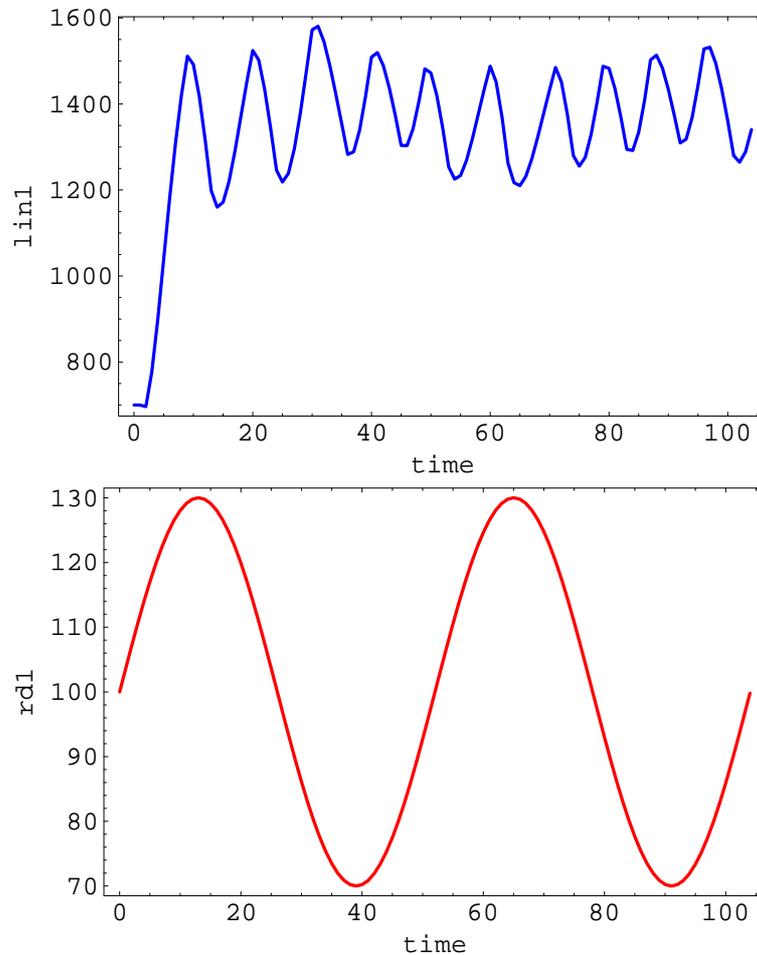


Figure 5. The characteristics of level of inventory of product  $P1$  ( $lin1$  – upper figure) and demand for this product ( $rd1$  – down figure) in whole horizon of simulation ( $lin1(0) = 700, lin2(0) = 700, sourc1 = 200, sourc2 = 350$ )

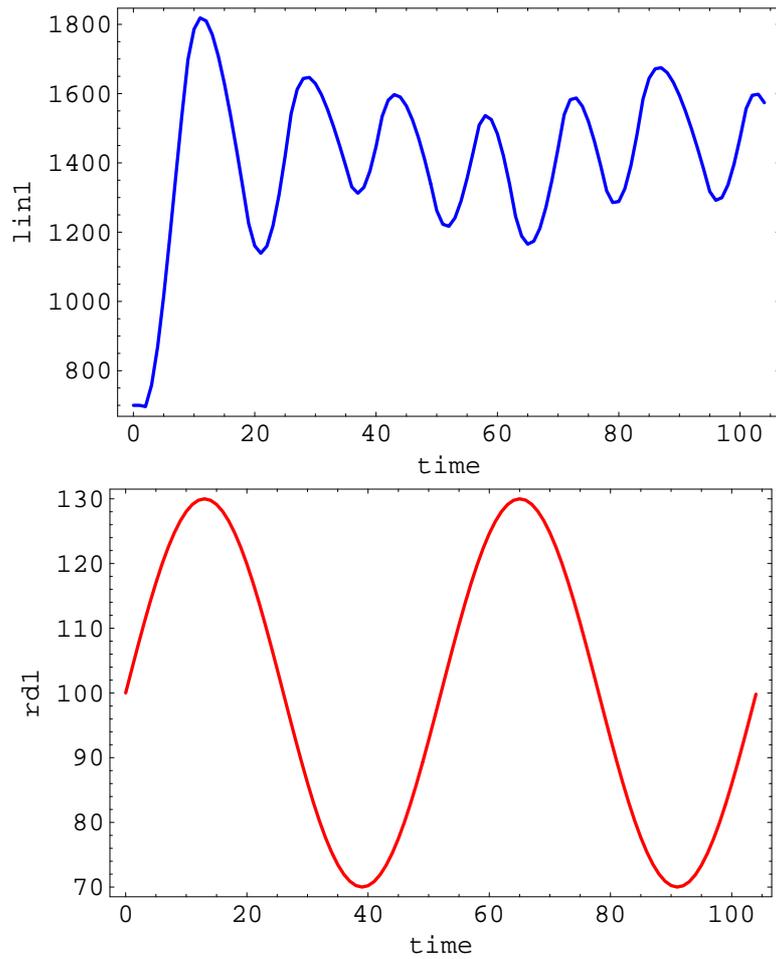


Figure 6. The characteristics of level of inventory of product  $P2$  ( $lin2$  – upper figure) and demand for this product ( $rd2$  – down figure) in whole horizon of simulation ( $lin1(0) = 700$ ,  $lin2(0) = 700$   $sourc1 = 200$ ,  $sourc2 = 350$ )

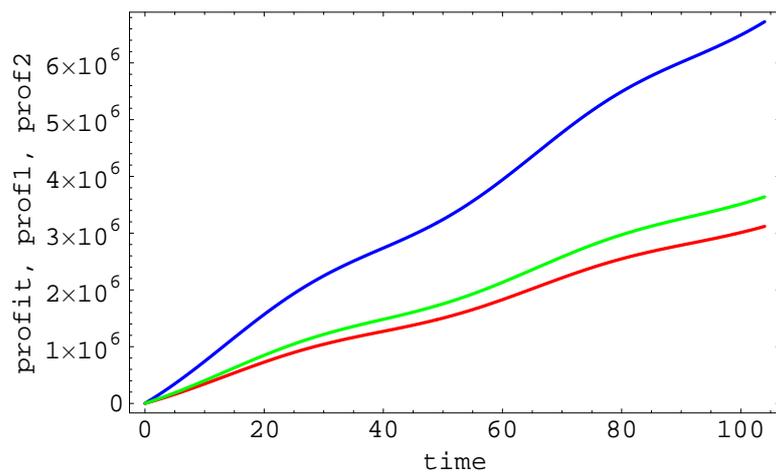


Figure 7. The characteristics of profits from sales of product  $P1$  ( $prof1$  – red line) and  $P2$  ( $prof2$  – green line) and profit of whole production ( $profit$  – blue line; taking into consideration the costs of production;  $lin1(0) = 700$ ,  $lin2(0) = 700$   $sourc1 = 200$ ,  $sourc2 = 350$ )

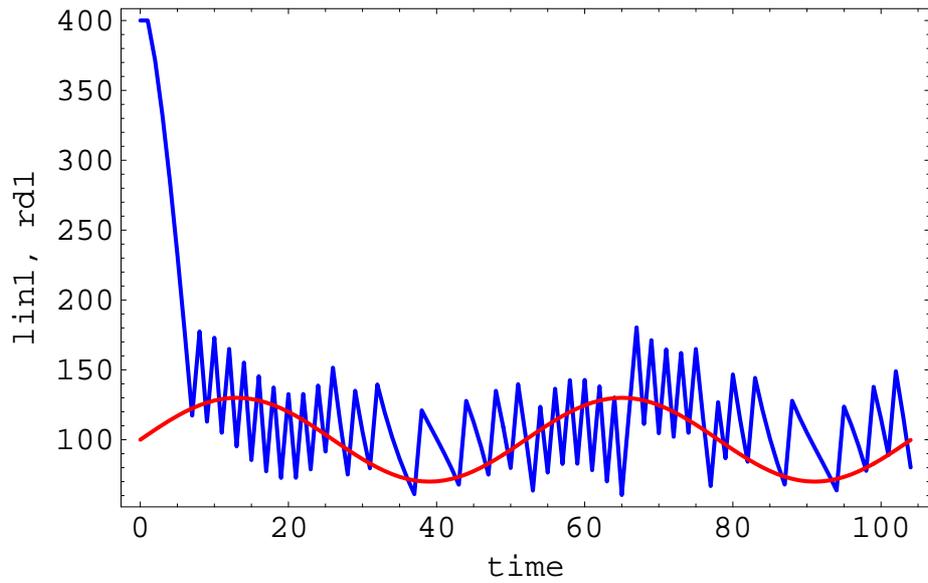


Figure 8. The characteristics of level of inventory of product  $P1$  ( $lin1$  – blue line) and demand for this product ( $rd1$  – red line) in whole horizon of simulation ( $lin1(0) = 400$ ,  $lin2(0) = 400$   $sourc1 = 20$ ,  $sourc2 = 20$ )

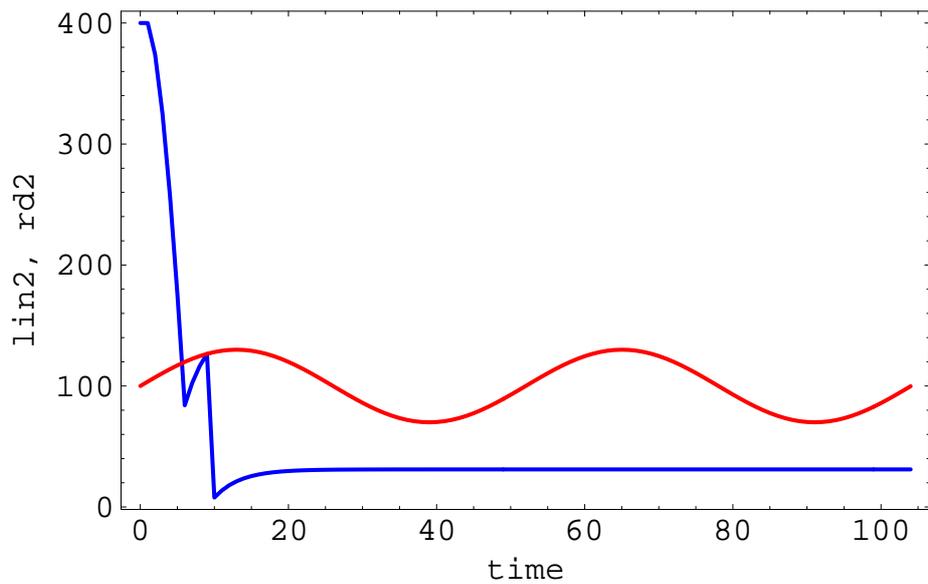


Figure 9. The characteristics of level of inventory of product  $P2$  ( $lin2$  – blue line) and demand for this product ( $rd2$  – red line) in whole horizon of simulation ( $lin1(0) = 400$ ,  $lin2(0) = 400$   $sourc1 = 20$ ,  $sourc2 = 20$ )

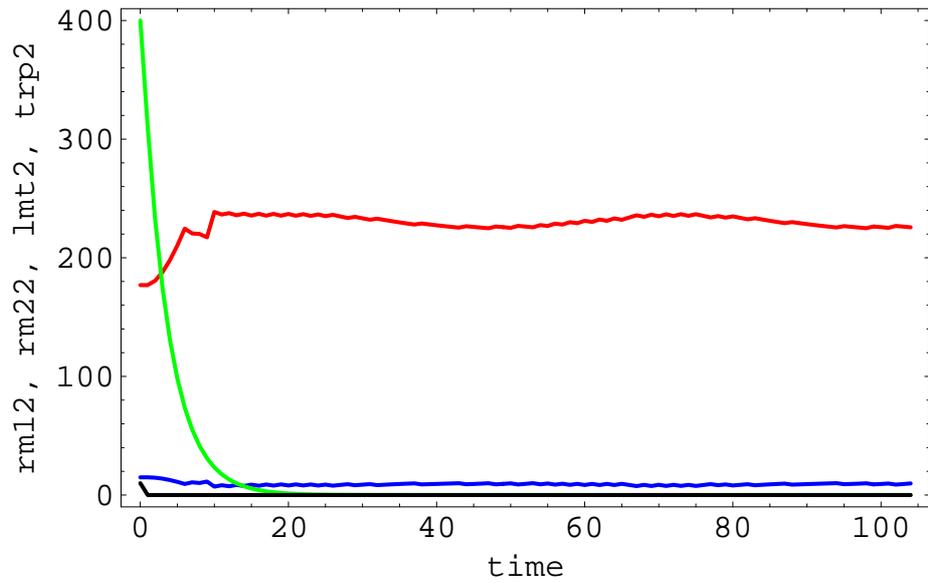


Figure 10. The dynamics of production of product  $P2$  under the condition of constraints of sources of raw materials ( $rm12$  – blue line;  $rm22$  – red line;  $lmt2$  – green line;  $trp2$  – black line)

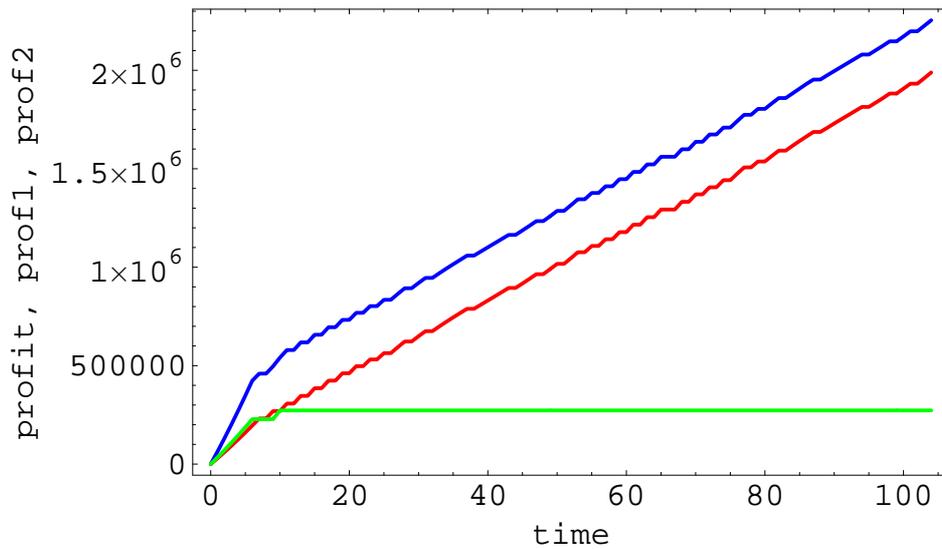


Figure 11. The dynamics of profits from sales of product  $P1$  ( $prof1$  – red line) and  $P2$  ( $prof2$  – green line) and profit of whole production ( $profit$  – blue line) under the condition of constraints of sources of raw materials (taking into consideration the costs of production)

Table 1. The fragment of "dynamics" of components of variables *norm* (the characteristic of *norm* we presented on Figure 3)

Time	<i>blb1</i>	<i>blb2</i>	<i>blb3</i>	<i>blb4</i>	<i>blb5</i>	<i>blb6</i>	<i>blb7</i>	<i>blb8</i>	<i>blb9</i>	<i>blb10</i>
0	-174.08	-72.74	-110.36	-68.23	207.51	133.32	-26.82	-83.54	1.37	80.40
1	-174.08	-72.74	-110.36	-68.23	207.51	133.32	-26.82	-83.54	1.37	80.40
2	-175.46	-73.72	-110.29	-67.34	208.30	133.80	-26.70	-83.59	1.67	80.99
3	-149.95	-58.92	-111.44	-81.77	193.16	126.72	-28.35	-83.08	-4.65	72.89
4	-111.04	-33.20	-113.37	-105.66	170.64	114.20	-31.54	-81.83	-13.53	57.86
5	-65.46	0.24	-115.83	-135.65	144.84	97.73	-35.97	-79.86	-23.10	37.45
6	-18.99	38.11	-118.56	-168.51	119.21	78.86	-41.28	-77.28	-31.94	13.42
7	24.35	77.37	-121.33	-201.53	96.02	59.12	-47.05	-74.28	-39.21	-12.32
8	62.04	115.30	-123.96	-232.53	76.53	39.87	-52.87	-71.09	-44.59	-37.95
9	92.79	149.66	-126.30	-259.89	61.22	22.31	-58.33	-67.97	-48.14	-61.74
10	88.94	165.79	-127.19	-268.79	66.79	13.33	-61.93	-65.27	-42.66	-76.13
11	67.02	169.13	-127.14	-266.09	82.66	10.64	-63.87	-63.27	-33.27	-82.82
12	35.55	163.33	-126.45	-255.81	103.56	12.56	-64.44	-62.01	-22.40	-83.42
13	-1.04	150.96	-125.31	-240.47	126.84	17.88	-63.91	-61.40	-11.14	-79.33
14	-15.32	136.63	-124.32	-228.75	134.24	25.14	-61.72	-62.60	-9.09	-69.66
15	-15.34	120.66	-123.39	-219.11	131.40	33.87	-58.37	-65.02	-13.02	-65.09
16	-4.74	103.57	-122.52	-211.07	120.97	43.64	-54.19	-68.33	-21.37	-39.70
17	12.30	85.60	-121.67	-203.90	105.88	54.14	-49.46	-72.21	-32.47	-21.44
18	32.21	66.92	-120.81	-196.92	88.68	65.14	-44.41	-76.40	-44.85	-2.06
19	52.08	47.63	-119.91	-189.56	71.38	76.48	-39.24	-80.67	-57.38	17.82
20	71.34	37.08	-119.52	-187.34	56.08	83.01	-35.94	-83.58	-67.50	30.16
21	64.08	32.09	-119.15	-182.76	60.25	85.45	-35.31	-83.84	-65.89	33.13
22	44.77	34.08	-119.04	-179.80	74.06	83.60	-36.82	-82.23	-57.86	28.05
23	20.56	43.05	-119.29	-180.00	92.53	77.73	-40.07	-79.22	-46.18	16.18
24	-5.11	57.99	-119.87	-183.49	113.09	68.55	-44.65	-75.21	-32.46	-1.03
25	-8.93	79.75	-121.09	-195.80	119.64	56.50	-49.43	-71.65	-25.61	-20.19
26	1.80	105.62	-122.71	-213.72	116.78	42.78	-54.25	-68.46	-23.43	-40.29
27	21.83	122.17	-123.90	-228.02	105.77	34.52	-56.58	-67.32	-27.18	-50.84
28	48.01	131.83	-124.76	-239.49	89.25	30.28	-57.13	-67.63	-35.03	-54.46
29	77.48	136.36	-125.36	-248.56	69.52	28.97	-56.41	-68.95	-45.44	-53.13
30	107.47	136.98	-125.73	-255.40	48.73	29.82	-54.84	-70.89	-57.01	-48.39
31	110.16	131.74	-125.46	-252.81	45.92	32.80	-53.58	-71.87	-59.35	-43.46
32	99.56	123.02	-124.83	-245.27	51.75	37.14	-52.36	-72.47	-57.36	-37.91
33	82.64	112.20	-124.01	-235.09	61.62	42.39	-51.05	-72.96	-53.41	-31.69
34	62.84	100.09	-123.08	-223.53	73.25	48.22	-49.63	-73.45	-48.65	-24.87
35	41.86	87.17	-122.09	-211.22	85.56	54.45	-48.11	-73.98	-43.63	-17.58
36	20.47	73.73	-121.07	-198.50	98.08	60.95	-46.51	-74.56	-38.58	-9.92
37	23.03	71.28	-120.96	-197.57	95.85	62.40	-45.85	-75.11	-40.19	-7.38
38	39.80	77.30	-121.50	-204.82	85.24	59.77	-46.16	-75.34	-45.26	-9.55
39	65.80	90.20	-122.54	-218.20	69.42	53.74	-47.39	-75.15	-52.25	-15.95
40	96.23	108.13	-123.93	-235.57	51.42	45.15	-49.43	-74.50	-59.72	-25.85

## 4 Final remarks and conclusions

The purpose of the paper was to present the two different methods of embedding the optimization in simulation on model DYNBALANCE(2-2). Authors have shown, using some mathematics, how formulate, in two ways, the problem of optimization embedded in System Dynamics model. The works with this subject are still in progress and it should be stress that many efforts waits for authors. So, it is time for the conclusions:

- authors have not shown in this paper the results of "working" the linear programming solver, because some difficulties have occurred in programming algorithm of linear programming in Dynamo language. But the conception is ready and authors present the mains elements of program in Appendix 2. We hopes, that the program of simulation of solving linear programming inside model DYNBALANCE(2-2) will be ready soon and we will be able to present the results to our Readers;
- possibilities of language Dynamo, makes possible to operate with such element like: "arrays", "macros" and with instruction "for". It is all very important in constructing the algorithm of calculating the "simplex tables" in linear programming program. It would be "easy job" in such languages like, for example "old good" DYSMAP (on big computers), which had possibilities to cooperate with Fortran (and of course, to use its rich library);
- when authors overcome the problem of minimizing using linear programming, it will be easy to ask a question of maximizing and authors hope that some dual results will occurs for interesting interpretation of the economic problem;
- the interesting point of view will be comparing the results of embedding the optimization in simulation on System Dynamics model with the classical Coyle's "simulation during optimization" (see (Coyle 1996, Coyle 1999)). Authors, partially, have undertaken this problem already in paper (Kasperska 2002a);
- the developing of the ideas of embedding optimization in simulation on System Dynamics models, enrich the possibilities of System Dynamics method, in the context of supporting the decision – makers in social and economic organization. So this effort lies in wide spectrum of methods of using artificial intelligence in support systems (precisely speaking: the using of hybrid models).

## References

- Coyle RG. 1996. *System Dynamics Modelling. A Practical Approach*. Chapman & Hall: London.
- Coyle RG. 1998. The practice System Dynamics: milestones, lessons and ideas from 30 years experience. *System Dynamics Rev.* **14**: 343–365.
- Coyle RG. 1999. Simulation by repeated optimisation. *J. Opt. R. S.* **50**: 429–438.

- Coyle RG. (ed.). 1994. *Cosmic and Cosmos. User manuals*. The Cosmic Holding Co: London.
- Coyle RG, Wolsterholm EP. 1980. Modelling discrete events in System Dynamics models – a case study. *Dynamica* **6**: 21–27.
- Forrester JW. 1961. *Industrial Dynamics*. MIT Press: Massachusetts.
- Forrester JW. 1969. *Urban Dynamics*. MIT Press: Massachusetts.
- Forrester JW. 1971. *World Dynamics*. Wright-Allen Press: Massachusetts.
- Forrester JW. 1972. *Principles of Systems*. Cambridge Press: Massachusetts.
- Forrester JW. 1975. *Collected papers of Jay W. Forrester*. Cambridge PressWright Allen Press: Massachusetts.
- Kasperska E. 1980. Analiza Dynamiki Systemów Zarządzania a metody optymalizacyjne w aspekcie potrzeb identyfikacji i konstrukcji procedur decyzyjnych. In: *Materiały Szkoły Symulacji Systemów Gospodarczych*. TNOIK Wrocław\IEPCh Gliwice: Lubachów, 23–27.
- Kasperska E. 1990. *Metodyka badań symulacyjnych dla wspomagania planowania i organizowania w przedsiębiorstwie przemysłowym o produkcji ciągłej*. Zakład Nauk Zarządzania PAN: Warszawa.
- Kasperska E. 2002a. Cybernetic formulation of some functions of management – types of simulation and optimization approaches within the System Dynamics method. In: P. I. Davidsen, E. Mollona, V. G. Diker, R. S. Langer and J. I. Rowe (eds). *Proc. 20 International Conference of the System Dynamics Society*. SDS: Palermo, 1–11.
- Kasperska E. 2002b. Supporting the decision in organization by the intelligent simulation package Cosmic and Cosmos. *Pr. Nauk. Akad. Ekonom. w Katowicach, "Support Systems in Organization"* 385–392.
- Kasperska E, Mateja-Losa E, Słota D. 2000a. Some extension of System Dynamics method – practical aspects. In: M. Deville and R. Owens (eds). *Proc. 16th IMACS World Congress*. IMACS: Lausanne, 718–11, 1–6.
- Kasperska E, Mateja-Losa E, Słota D. 2000b. Some extension of System Dynamics method – theoretical aspects. In: M. Deville and R. Owens (eds). *Proc. 16th IMACS World Congress*. IMACS: Lausanne, 718–10, 1–6.
- Kasperska E, Mateja-Losa E, Słota D. 2001. Some dynamics balance of production via optimization and simulation within System Dynamics method, In: J. H. Hines, V. G. Diker, R. S. Langer and J. I. Rowe (eds). *Proc. 19th International Conference of the System Dynamics Society*. SDS: Atlanta, 1–18.

- Kasperska E, Mateja-Losa E, Słota D. 2002. Optimal dynamical balance of raw materials – some concept of embedding optimization in simulation on system dynamics models and vice versa. *In: P. I. Davidsen, E. Mollona, V. G. Diker, R. S. Langer and J. I. Rowe (eds). Proc. 20 International Conference of the System Dynamics Society.* SDS: Palermo, 1–23.
- Kasperska E, Słota D. 2000. *Metody matematyczne w zarządzaniu w ujęciu Dynamiki Systemowej, wyd. II.* Wyd. Pol. Śl.: Gliwice.
- Keloharju R. 1977. System Dynamics or Super Dynamics. *Dynamica* 4, 26–43.
- Legras J. 1974. *Praktyczne metody analizy numerycznej.* WNT: Warszawa.
- Łukaszewicz R. 1975. *Dynamika systemów zarządzania.* PWN: Warszawa.
- Łukaszewicz R. 1976. The direct form of structure models within System Dynamics. *Dynamica* 2.
- Pugh AL (ed.). 1994. *Professional Dynamo 4.0 for Windows. Tutorial Guide.* Pugh-Roberts Associates: Cambridge.
- Senge PH. 1998. *Piąta dyscyplina.* Dom Wyd. ABC: Warszawa.
- Senge PH. 2002. *Piąta dyscyplina zbiór ćwiczeń.* Oficyna Ekonomiczna: Warszawa.
- Sterman JD. 2000. *Business dynamics – system thinking and modeling for a complex world.* Mc Graw-Hill: Boston.

## Appendix 1. Program in Dynamo

```
* Balance of two raw materials and two products
note
note level of raw materials during transformation into product p1
note
n lmt1=200
l lmt1.k=lmt1.j+dt*(ttrp1.j-rpr1.jk)
a ttrp1.k=clip(trp1.k,0,desir1,lin1.k)
note
note transformation of raw materials into product p1
note
a trp1.k=q11*prrm11.k+q12*prrm21.k
note
note rate of production of product p1
note
r rpr1.kl=switch(0,lmt1.k/tpr1,lmt1.k)
note
note gains of transformation raw materials into product p1
note
c q11=1
```

```

c q12=2
note
note time of transformation in lmt1
note
c tpr1=2
note
note level of raw materials during transformation into product p2
note
n lmt2=400
l lmt2.k=lmt2.j+dt*(ttrp2.j-rpr2.jk)
a ttrp2.k=clip(trp2.k,0,desir2,lin2.k)
note
note transformation of raw materials into product p2
note
a trp2.k=q21*prrm12.k+q22*prrm22.k
note
note rate of production of product p2
note
r rpr2.k1=switch(0,lmt2.k/tpr2,lmt2.k)
note
note gains of transformation raw materials into product p2
note
c q21=1
c q22=2
note
note time of transformation in lmt2
note
c tpr2=4
note
note
note level of inventory of product p1
note
n lin1=350
l lin1.k=lin1.j+dt*(rpr1.jk-rs11.j)
note
note sale of product p1
note
a rs11.k=clip(0,rd1.k,rd1.k*dt,lin1.k)
note
note demand for product p1
note
a rd1.k=po+p1*sin((6.28*time.k)/perd)
c po=100
c p1=30
c perd=52
note

```

```

note indicator of the losse of sale of product p1
note
a ilosl1.k=switch(1,0,rs11.k)
note
note level of the lossen sale of product p1
note
n los11=0
l los11.k=los11.j+dt*(rd1.j-rs11.j)
note
note losse of profit from potencial sale of product p1
note
a loprf1.k=los11.k*price1
note
note
note level of inventory of product p2
note
n lin2=350
l lin2.k=lin2.j+dt*(rpr2.jk-rs12.j)
note
note sale of product p2
note
a rs12.k=clip(0,rd2.k,rd2.k*dt,lin2.k)
note
note demand for product p2
note
a rd2.k=po+p2*sin((6.28*time.k)/perd)
c p2=30
note
note indicator of the losse of sale of product p2
note
a ilosl2.k=switch(1,0,rs12.k)
note
note level of the lossen sale of product p2
note
n los12=0
l los12.k=los12.j+dt*(rd2.j-rs12.j)
note
note losse of profit from potencial sale of product p2
note
a loprf2.k=los12.k*price2
note
note prices of products p1 and p2 on market
note
c price1=300
c price2=350
note

```

note costs of production of products p1 and p2  
 note  
 $a \text{ copr1.k} = \text{ucp11} * \text{q11} * \text{prrm11.k} + \text{ucp21} * \text{q12} * \text{prrm21.k}$   
 $a \text{ copr2.k} = \text{ucp12} * \text{q21} * \text{prrm12.k} + \text{ucp22} * \text{q22} * \text{prrm22.k}$   
 note  
 note levels of costs of productions of products p1 and p2  
 note  
 $n \text{ lcopr1} = 0$   
 $l \text{ lcopr1.k} = \text{lcopr1.j} + dt * \text{copr1.j}$   
 $n \text{ lcopr2} = 0$   
 $l \text{ lcopr2.k} = \text{lcopr2.j} + dt * \text{copr2.j}$   
 note  
 note levels of sales of the product p1 and p2  
 note  
 $n \text{ lsale1} = 0$   
 $l \text{ lsale1.k} = \text{lsale1.j} + dt * \text{rs11.j}$   
 $n \text{ lsale2} = 0$   
 $l \text{ lsale2.k} = \text{lsale2.j} + dt * \text{rs12.j}$   
 note  
 note profit from production  
 note  
 $a \text{ profit.k} = \text{prof1.k} - \text{lcopr1.k} + \text{prof2.k} - \text{lcopr2.k}$   
 $a \text{ prof1.k} = \text{lsale1.k} * \text{price1}$   
 $a \text{ prof2.k} = \text{lsale2.k} * \text{price2}$   
 note  
 note  
 note  
 $c \text{ ucp1} = 10$   
 $c \text{ ucp2} = 5$   
 $c \text{ ucp3} = 2$   
 note  
 note unit cost of production of product p1 from raw materials  
 note  
 $c \text{ ucp11} = 1.50$   
 $c \text{ ucp21} = 1.00$   
 note  
 note unit cost of production of product p2 from raw materials  
 note  
 $c \text{ ucp12} = 2.00$   
 $c \text{ ucp22} = 0.5$   
 note  
 note forecasted rate of demand for product p1 (plan of production of product p1)  
 note  
 $a \text{ frdp1.k} = \text{ard1.k} + (\text{desir1} - \text{lin1.k}) / \text{tau1}$   
 note  
 note averaging rate of demand for product p1

```

note
n ard1=100
l ard1.k=ard1.j+dt*(rd1.j-ard1.j)/tasmo1
note
note time of smoothing the demand for product p1
note
c tasm01=12
note
note desired value of level of inventory of product p1
note
c desir1=1400
note
note time of adjusting of inventory lin1 to the desired value
note
c tau1=2
note
note forecasted rate of demand for product p2 (plan of production of product p2)
note
a frdp2.k=ard2.k+(desir2-lin2.k)/tau2
note
note averaging rate of demand for product p2
note
n ard2=100
l ard2.k=ard2.j+dt*(rd2.j-ard2.j)/tasmo2
note
note time of smoothing the demand for product p2
note
c tasm02=12
note
note desired value of level of inventory of product p2
note
c desir2=1400
note
note time of adjusting of inventory lin2 to the desired value
note
c tau2=2
note
note characteristics of sources of raw materials
note
a sourc1.k=20
a sourc2.k=20
note
note rrm11 - rate of raw material (rm11)
note
r rrm11.kl=clip(init11,arm11.k,0,time.k)
a id11.k=1

```

```

l arm11.k=arm11.j+dt*(rm11.jk-arm11.j)/at11
n arm11=init11
c at11=1
c init11=10
note
note rrm12 - rate of raw material (rm12)
note
r rrm12.kl=clip(init12,arm12.k,0,time.k)
a id12.k=1
l arm12.k=arm12.j+dt*(rm12.jk-arm12.j)/at12
n arm12=init12
c at12=1
c init12=20
note
note rrm21 - rate of raw material (rm21)
note
r rrm21.kl=clip(init21,arm21.k,0,time.k)
a id21.k=1
l arm21.k=arm21.j+dt*(rm21.jk-arm21.j)/at21
n arm21=init21
c at21=1
c init21=45
note
note rrm22 - rate of raw material (rm22)
note
r rrm22.kl=clip(init22,arm22.k,0,time.k)
a id22.k=1
l arm22.k=arm22.j+dt*(rm22.jk-arm22.j)/at22
n arm22=init22
c at22=1
c init22=40
note
note prrm11 - proofed rate of raw material rm11
note prrm12 - proofed rate of raw material rm12
note prrm21 - proofed rate of raw material rm21
note prrm22 - proofed rate of raw material rm22
note
a prrm11.k=clip(0,amin11.k,0,amin11.k)
a amin11.k=min(rrm11.kl,sourc1.k)
a prrm12.k=clip(0,amin12.k,0,amin12.k)
a amin12.k=min(min(rrm12.kl,sourc1.k-rrm11.kl),sourc1.k)
a prrm21.k=clip(0,amin21.k,0,amin21.k)
a amin21.k=min(rrm21.kl,sourc2.k)
a prrm22.k=clip(0,amin22.k,0,amin22.k)
a amin22.k=min(min(rrm22.kl,sourc2.k-rrm21.kl),sourc2.k)
note

```

```
note vector b
note
a b1.k=frdp1.k
a b2.k=frdp2.k
a b3.k=sourc1.k
a b4.k=sourc2.k
a b5.k=prf1b5.k+prf2b5.k
a b6.k=prf1b6.k+prf2b6.k
a b7.k=100
a b8.k=100
a b9.k=100
a b10.k=100
note
note
a prf1b5.k=clip(50000,15,0,arm11.k)
a prf2b5.k=clip(50000,90,0,arm12.k)
a prf1b6.k=clip(50000,40,0,arm21.k)
a prf2b6.k=clip(50000,40,0,arm22.k)
note
note some elements of matrix a
note
a a11.k=q11
a a12.k=0
a a13.k=q12
a a14.k=0
a a21.k=0
a a22.k=q21
a a23.k=0
a a24.k=q22
a a31.k=1
a a32.k=1
a a33.k=0
a a34.k=0
a a41.k=0
a a42.k=0
a a43.k=1
a a44.k=1
a a51.k=ucp11*q11
a a52.k=0
a a53.k=ucp21*q12
a a54.k=0
a a61.k=0
a a62.k=ucp12*q21
a a63.k=0
a a64.k=ucp22*q22
a a71.k=id11.k
```

```

a a72.k=0
a a73.k=0
a a74.k=0
a a81.k=0
a a82.k=id12.k
a a83.k=0
a a84.k=0
a a91.k=0
a a92.k=0
a a93.k=id21.k
a a94.k=0
a a101.k=0
a a102.k=0
a a103.k=0
a a104.k=id22.k
note
note vector at.b
note
a bb1.k=a11.k*b1.k+a101.k*b10.k+a21.k*b2.k+a31.k*b3.k+a41.k*b4.k+a51.k*b5.k+^
a61.k*b6.k+a71.k*b7.k+a81.k*b8.k+a91.k*b9.k
a bb2.k=a12.k*b1.k+a102.k*b10.k+a22.k*b2.k+a32.k*b3.k+a42.k*b4.k+a52.k*b5.k+^
a62.k*b6.k+a72.k*b7.k+a82.k*b8.k+a92.k*b9.k
a bb3.k=a13.k*b1.k+a103.k*b10.k+a23.k*b2.k+a33.k*b3.k+a43.k*b4.k+a53.k*b5.k+^
a63.k*b6.k+a73.k*b7.k+a83.k*b8.k+a93.k*b9.k
a bb4.k=a14.k*b1.k+a104.k*b10.k+a24.k*b2.k+a34.k*b3.k+a44.k*b4.k+a54.k*b5.k+^
a64.k*b6.k+a74.k*b7.k+a84.k*b8.k+a94.k*b9.k
note
note matrix c=at.a
note
a c11.k=a101.k*a101.k+a11.k*a11.k+a21.k*a21.k+a31.k*a31.k+a41.k*a41.k+^
a51.k*a51.k+a61.k*a61.k+a71.k*a71.k+a81.k*a81.k+a91.k*a91.k
a c12.k=a101.k*a102.k+a11.k*a12.k+a21.k*a22.k+a31.k*a32.k+a41.k*a42.k+^
a51.k*a52.k+a61.k*a62.k+a71.k*a72.k+a81.k*a82.k+a91.k*a92.k
a c13.k=a101.k*a103.k+a11.k*a13.k+a21.k*a23.k+a31.k*a33.k+a41.k*a43.k+^
a51.k*a53.k+a61.k*a63.k+a71.k*a73.k+a81.k*a83.k+a91.k*a93.k
a c14.k=a101.k*a104.k+a11.k*a14.k+a21.k*a24.k+a31.k*a34.k+a41.k*a44.k+^
a51.k*a54.k+a61.k*a64.k+a71.k*a74.k+a81.k*a84.k+a91.k*a94.k
a c21.k=a101.k*a102.k+a11.k*a12.k+a21.k*a22.k+a31.k*a32.k+a41.k*a42.k+^
a51.k*a52.k+a61.k*a62.k+a71.k*a72.k+a81.k*a82.k+a91.k*a92.k
a c22.k=a102.k*a102.k+a12.k*a12.k+a22.k*a22.k+a32.k*a32.k+a42.k*a42.k+^
a52.k*a52.k+a62.k*a62.k+a72.k*a72.k+a82.k*a82.k+a92.k*a92.k
a c23.k=a102.k*a103.k+a12.k*a13.k+a22.k*a23.k+a32.k*a33.k+a42.k*a43.k+^
a52.k*a53.k+a62.k*a63.k+a72.k*a73.k+a82.k*a83.k+a92.k*a93.k
a c24.k=a102.k*a104.k+a12.k*a14.k+a22.k*a24.k+a32.k*a34.k+a42.k*a44.k+^
a52.k*a54.k+a62.k*a64.k+a72.k*a74.k+a82.k*a84.k+a92.k*a94.k
a c31.k=a101.k*a103.k+a11.k*a13.k+a21.k*a23.k+a31.k*a33.k+a41.k*a43.k+^

```



$c43.k+c13.k*c21.k*c44.k-c11.k*c23.k*c44.k$   
a  $d24.k=-c14.k*c23.k*c31.k+c13.k*c24.k*c31.k+c14.k*c21.k*c33.k-c11.k*c24.k*c33.k-c13.k*c21.k*c34.k+c11.k*c23.k*c34.k$   
a  $d31.k=-c24.k*c32.k*c41.k+c22.k*c34.k*c41.k+c24.k*c31.k*c42.k-c21.k*c34.k*c42.k-c22.k*c31.k*c44.k+c21.k*c32.k*c44.k$   
a  $d32.k=c14.k*c32.k*c41.k-c12.k*c34.k*c41.k-c14.k*c31.k*c42.k+c11.k*c34.k*c42.k+c12.k*c31.k*c44.k-c11.k*c32.k*c44.k$   
a  $d33.k=-c14.k*c22.k*c41.k+c12.k*c24.k*c41.k+c14.k*c21.k*c42.k-c11.k*c24.k*c42.k-c12.k*c21.k*c44.k+c11.k*c22.k*c44.k$   
a  $d34.k=c14.k*c22.k*c31.k-c12.k*c24.k*c31.k-c14.k*c21.k*c32.k+c11.k*c24.k*c32.k+c12.k*c21.k*c34.k-c11.k*c22.k*c34.k$   
a  $d41.k=c23.k*c32.k*c41.k-c22.k*c33.k*c41.k-c23.k*c31.k*c42.k+c21.k*c33.k*c42.k+c22.k*c31.k*c43.k-c21.k*c32.k*c43.k$   
a  $d42.k=-c13.k*c32.k*c41.k+c12.k*c33.k*c41.k+c13.k*c31.k*c42.k-c11.k*c33.k*c42.k-c12.k*c31.k*c43.k+c11.k*c32.k*c43.k$   
a  $d43.k=c13.k*c22.k*c41.k-c12.k*c23.k*c41.k-c13.k*c21.k*c42.k+c11.k*c23.k*c42.k+c12.k*c21.k*c43.k-c11.k*c22.k*c43.k$   
a  $d44.k=c13.k*c22.k*c31.k+c12.k*c23.k*c31.k+c13.k*c21.k*c32.k-c11.k*c23.k*c32.k-c12.k*c21.k*c33.k+c11.k*c22.k*c33.k$

note

note rates of raw materials for production of products p1 and p2

note

$r\ rm11.kl=(bb1.k*d11.k+bb2.k*d12.k+bb3.k*d13.k+bb4.k*d14.k)/detc.k$   
 $r\ rm12.kl=(bb1.k*d21.k+bb2.k*d22.k+bb3.k*d23.k+bb4.k*d24.k)/detc.k$   
 $r\ rm21.kl=(bb1.k*d31.k+bb2.k*d32.k+bb3.k*d33.k+bb4.k*d34.k)/detc.k$   
 $r\ rm22.kl=(bb1.k*d41.k+bb2.k*d42.k+bb3.k*d43.k+bb4.k*d44.k)/detc.k$

note

note

note

a  $b11.k=(q11*rm11.kl+q12*rm21.kl-b1.k)*(q11*rm11.kl+q12*rm21.kl-b1.k)$   
a  $b12.k=(q21*rm12.kl+q22*rm22.kl-b2.k)*(q21*rm12.kl+q22*rm22.kl-b2.k)$   
a  $b13.k=(rm11.kl+rm12.kl-b3.k)*(rm11.kl+rm12.kl-b3.k)$   
a  $b14.k=(rm21.kl+rm22.kl-b4.k)*(rm21.kl+rm22.kl-b4.k)$   
a  $b15.k=(a51.k*rm11.kl+a53.k*rm21.kl-b5.k)*(a51.k*rm11.kl+a53.k*rm21.kl-b5.k)$   
a  $b16.k=(a62.k*rm12.kl+a64.k*rm22.kl-b6.k)*(a62.k*rm12.kl+a64.k*rm22.kl-b6.k)$   
a  $b17.k=(rm11.kl-b7.k)*(rm11.kl-b7.k)$   
a  $b18.k=(rm12.kl-b8.k)*(rm12.kl-b8.k)$   
a  $b19.k=(rm21.kl-b9.k)*(rm21.kl-b9.k)$   
a  $b110.k=(rm22.kl-b10.k)*(rm22.kl-b10.k)$   
a  $norm.k=\sqrt{b11.k+b12.k+b13.k+b14.k+b15.k+b16.k+b17.k+b18.k+b19.k+b110.k}$

note

note

a  $blb1.k=(q11*rm11.kl+q12*rm21.kl-b1.k)$   
a  $blb2.k=(q21*rm12.kl+q22*rm22.kl-b2.k)$   
a  $blb3.k=(rm11.kl+rm12.kl-b3.k)$   
a  $blb4.k=(rm21.kl+rm22.kl-b4.k)$

```

a blb5.k=(a51.k*rm11.kl+a53.k*rm21.kl-b5.k)
a blb6.k=(a62.k*rm12.kl+a64.k*rm22.kl-b6.k)
a blb7.k=(rm11.kl-b7.k)
a blb8.k=(rm12.kl-b8.k)
a blb9.k=(rm21.kl-b9.k)
a blb10.k=(rm22.kl-b10.k)
note
note parameters of simulation
note
spec length=104/dt=1/savper=1
save rm11,rm21,lmt1,trp1,rm12,rm22,lmt2,trp2,lin1,rpr1,rsl1,rd1,lin2,rpr2,rsl2
save rd2,ilos11,los11,loprf1,ilos12,los12,loprf2,copr1,copr2,lcopr1,lcopr2
save frdp1,ard1,frdp2,ard2,norm,blb1,blb2,blb3,blb4,blb5,blb6,blb7,blb8,blb9
save blb10,rrm11,rrm12,rrm21,rrm22,arm11,arm12,arm21,arm22,prrm11,prrm12
save prrm21,prrm22,id11,id12,id21,id22,profit,prof1,prof2

```

## Appendix 2. Elements of program of linear programming solver for model DYNBALANCE(2-2-LP)

### I. Declaration of matrix $a$ (first simplex table):

```

t a(*,1)=frdp1.k,1,2,0,0,-1,0,0,0,1,0
t a(*,2)=frdp2.k,0,0,1,2,0,-1,0,0,0,1
t a(*,3)=sourc1.k,1,0,1,0,0,0,1,0,0,0
t a(*,4)=sourc2.k,0,1,0,1,0,0,0,1,0,0
t a(*,5)=a50.k,a51.k,a52.k,a53.k,a54.k,a55.k,^
a56.k,a57.k,a58.k,a59.k,a510.k

```

### II. Calculation the lambda for which:

```

lambda=min(a50.k,a51.k,a52.k,a53.k,a54.k,a55.k,^
a56.k,a57.k,a58.k,a59.k,a510.k)

```

It can be realising by using instruction min in way:

```

lambda=min(a50.k,min(a51.k,min(a52.k,min(a53.k,min(a54.k,min(a55.k,^
min(a56.k,min(a57.k,min(a58.k,min(a59.k,a510.k))))))))))

```

### III. Identification of index $k$ for which:

```
a5k.k=lambda.k
```

It can be made in "cascading" way:

```
idenfk.k=clip(1,id1k.k,lambdak.k,a51.k)
id1k.k=clip(2,id2k.k,lambdak.k,a52.k)
id2k.k=clip(3,id3k.k,lambdak.k,a53.k)
id3k.k=clip(4,id4k.k,lambdak.k,a54.k)
id4k.k=clip(5,id5k.k,lambdak.k,a55.k)
id5k.k=clip(6,id6k.k,lambdak.k,a56.k)
id6k.k=clip(7,id7k.k,lambdak.k,a57.k)
id7k.k=clip(8,id8k.k,lambdak.k,a58.k)
id8k.k=clip(9,10,lambdak.k,a59.k)
```

Of course, in final program, authors replace this "cascading" instruction by appropriate "macro" instruction.

IV. Identification of index  $l$ , element in "simplex table" named  $x_{kl}$  will be so called "solving element". The variable `idenfl` will be represented the searched index  $l$ :

```
idenfl.k=clip(1,id1l.k,alfak.k,cb1.k/a1k)
id1l.k=clip(2,id2l.k,alfak.k,cb2.k/a2k)
id2l.k=clip(3,4,alfak.k,cb3.k/a3k)

alfak.k=min(cb1.k/a1k,min(cb2.k/a2k,min(cb3.k/a3k,cb4.k/a4k)))
```

It should be determined the table  $c$  (first table for price component of minimizing function  $f(x) = cx$ ):

```
t c(*,1)=c1,c2,c3,c4,c5,c6,c7,c8,c9,c10
```

In first step of working simplex algorithm the variables `cb1`, `cb2`, `cb3` and `cb4` are determined:

```
cb1.k=c9
cb2.k=c10
cb3.k=0
cb4.k=0
```

Generally, in final program, should be defined of the elements the table `ncb`, in way:

```
for i=1,4
ncb.k(i)=switch(c(idenfkk.k),cb(i),i-idenfl.k)
```

V. When we have identified the position of "solving" element, it means: `idenfk`, `idenfl`, we can determine the second step of "simplex table" named `a2`:

```
for i=1,5
for j=0,10
a2.k(i,j)=switch(aa2.k(i,j),ba2(i,j),i-idenfl.k)

for j=0,10
aa2.k(idenfl.k,j)=a.k(idenfl.k,j)/a.k(idenfl.k,idenfl.k)

for i=1,5
for j=0,10
ba2.k(i,j)=a.k(i,j)-a.k(idenfl.k,j)*a.k(i,idenfk.k)/a.k(idenfl.k,idenfk.k)
```

VI. The remaining step of algorithm (authors should work on it) will be identifying the values of variables:  $x_1$ ,  $x_2$ , ...,  $x_{10}$  in successive steps of transforming the "simplex table". Lets notice, that in first step:

```
x9=frdp1(0)
x10=frdp2(0)
x7=sourc1(0)
x8=sourc2(0)
x1=x2=x3=x4=x5=x6=0
```

Lets notice too, that the final values of variable  $x_i$ ,  $i = 1, 2, \dots, 10$  (it mean, the values in final step of transforming the "simplex table") will be the value of final "simplex table" on its first column.

Authors hopes that the presentation of above mentioned elements, will help the Readers to learn about the algorithm of linear programming (in Dynamo language).